Proper generalized decomposition for parameterized elliptic PDE's

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ABSTRACT

The computational cost of parametric studies currently represents the major limitation to the application of simulation-based engineering techniques in a daily industrial environment. Parameters of interest in such analyses include shape of the domain, material properties and boundary conditions. It is well-known that the computational complexity of approximating the partial differential equations (PDE's) describing these problems increases exponentially with the number of parameters considered. To handle the resulting high-dimensional parameterized PDE's, the proper generalized decomposition (PGD) rationale is exploited [1]. The construction of explicit parametric solutions is discussed using low and high-order hybridizable discontinuous Galerkin (HDG) approaches for the spatial problems [2]. On the one hand, fast and robust computations are provided by the lowest order HDG approximation, the recently proposed face-centered finite volume method [3, 4]. On the other hand, high-fidelity solutions are obtained by means of high-order HDG discretizations with non-uniform and adaptive polynomial degree of approximation [5, 6, 7]. Numerical examples will be presented as a proof-of-concept for the application of PGD to the solution of parameterized elliptic PDE's of industrial interest.

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