Moving Mesh Algorithms for Time-Dependent Hyperbolic Conservation Laws

M. E. Hubbard*, M. Ricchiuto[1] and D. Sármány[2]

* School of Mathematical Sciences, University of Nottingham
University Park, Nottingham, NG7 2RD, UK
e-mail: matthew.hubbard@nottingham.ac.uk

[1] Team CARDAMOM, INRIA Bordeaux – Sud-Ouest
200 Avenue de la Vieille Tour, 33405 Talence Cedex, France
e-mail: mario.ricchiuto@inria.fr

[2] European Centre for Medium Range Weather Forecasts
Shinfield, Reading, Berkshire, RG2 9AX, UK
e-mail: domokos.sarmany@ecmwf.int

ABSTRACT

This talk will assess the potential for a moving mesh algorithm to enhance the efficiency of space-time schemes for approximating systems of time-dependent hyperbolic conservation laws [1]. The connectivity of the mesh, and hence the number of degrees of freedom, remain fixed, so there should be no detrimental effect on parallel implementations, but the mesh nodes are moved as the solution evolves so that features of the solution remain resolved as they move within the domain.

In particular, this adaptive strategy will be applied to residual distribution schemes which, when discontinuities are allowed in time, can be designed to be positive (and hence stable) for any choice of time-step [2], even on the distorted space-time prisms which arise from moving the nodes of an unstructured triangular mesh [1]. Consequently, a local increase in mesh resolution does not impose a more restrictive stability constraint on the time-step, which can instead be chosen according to accuracy requirements. The link between these space-time schemes and conservative ALE formulations which automatically satsify the discrete geometric conservation law (dictating that the mesh movement will not artificially create or destroy volume) will be outlined.

Numerical results will be shown in two space dimensions for both scalar advection and shallow water flows over variable bed topography. In the latter case, it is shown that these moving mesh schemes can be designed so that they satisfy the so-called C-property (retention of hydrostatic balance between flux and source terms, required to maintain the steady state of still, flat, water). Even for a very simple implementation of the mesh movement (driven by a monitor function based on the scaled gradient and Laplacian of the dependent variables), reductions in cpu time of up to 60% have been attained without increasing the error.

REFERENCES

[1] Hubbard, M.E., Ricchiuto, M. and Sármány D. *Space-time residual distribution on moving meshes*, submitted to Comput Math Appl, 2018.

[2] Sármány, D., Hubbard, M.E. and Ricchiuto, M. *Unconditionally stable space-time discontinuous residual distribution for shallow-water flows*, J Comput Phys 253:86–113, 2013.