An adaptive method for the numerical approximation of orthogonal maps

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ABSTRACT

Orthogonal maps are the solutions of the analytical model of paper-folding, also called origami problem [1]. They consist of a system of first order fully nonlinear equations involving the gradient of the solution. The Dirichlet problem for orthogonal maps is considered here, namely find a vector-valued function $\mathbf{u} : \Omega \subset \mathbb{R}^2 \to \mathbb{R}^2$ verifying

$$\begin{cases} \nabla \mathbf{u} \in \mathcal{O}(2) & \text{in } \Omega, \\ \mathbf{u} = \mathbf{g} & \text{on } \partial\Omega. \end{cases}$$

where $\mathcal{O}(2)$ denotes the set of orthogonal matrix-valued functions, and $\mathbf{g}$ is a given, sufficiently smooth, function. The solution to this problem is piecewise linear, with a singular set composed of straight lines representing the folding edges.

A variational approach relies on the minimization of a variational principle, which enforces the uniqueness of the solution. We present a strategy based on a splitting algorithm for the flow problem derived from the first-order optimality conditions. It leads to decoupling the time-dependent problem into a sequence of local nonlinear problems and a global variational problem at each time step.

Within the splitting algorithm, adaptive techniques are introduced and rely on error estimate based techniques developed for the solution of linear Poisson problems [2]. Anisotropic mesh adaptivity allows to obtain unstructured triangulations that track the line discontinuities of the gradient of the solution.

Numerical experiments validate the accuracy and the efficiency of the method for various domains and meshes. Adaptive meshes show appropriate convergence properties, and allow to recover solutions with sharp edges.

REFERENCES

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