Adaptive moving mesh finite difference methods for PDEs using the Monge-Ampére equation

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ABSTRACT

We present a moving mesh finite difference method for the solution of PDEs on rectangular, convex and non-convex curved domains. We start by extending the Parabolic Monge-Ampére methods (PMA) developed by Budd and Williams [1] to solve a fourth order nonlinear PDE with finite time singularities and propagating interfaces. A key feature in our implementation is the generation of a high order transformation between computational and physical meshes that can accommodate the high order derivatives in the PDE [3].

Accurately resolving singularities on general domains motivates recent work in extending the Parabolic Monge-Ampére equation to problems with curved domains. We utilize the optimal transport boundary formulation from [2, 4] to create a mapping between a fixed computational domain, on which all derivative calculations are made, to a curved physical domain. This creates an initial mesh mapping which can be paired to the PMA to adaptively resolve fine-scale features in the PDE. We give results of this method for a variety of examples including semi-linear blow-up, sharp interfaces and prescribed boundary motion on convex and select non-convex domains.

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