From an efficient a posteriori algebraic error estimator to a p-robust multilevel solver

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ABSTRACT

In this work, we consider conforming finite element discretizations of the Poisson equation of arbitrary polynomial degree p. We propose a multilevel iterative algebraic solver, as well as an a posteriori estimator on the algebraic error. Firstly, we show that the iterative solver contracts the algebraic error on each iteration with a factor bounded independently of the degree p. Secondly, we prove that the algebraic error estimate is efficient (represents a two-sided bound of the error), again with a constant independent of p. Actually, we show that these two results are equivalent. The p-robustness results rely on the work of [2] for one given mesh and on the design of an algebraic residual lifting constructed over a hierarchy of meshes, in the spirit of [1]. The construction of the algebraic residual lifting is obtained by employing one multigrid-type iteration to approximate the algebraic error. This includes a global coarse level piecewise affine approximation and local patchwise high-order contributions emerging in the subsequent mesh levels. The high-order contributions are given as solutions to local Dirichlet problems which are mutually independent and thus naturally parallelizable. This residual lifting is the core of our a posteriori estimator and determines the descent direction for the next iteration of the multilevel solver. Numerical tests are presented to illustrate these theoretical findings.

REFERENCES

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