PDE-based quad meshing: accuracy through p-adaptation

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ABSTRACT

Recent research efforts in quad meshing have focused on cross-field based approaches [1, 2], initially inspired by the computer graphics community. Cross-fields are useful in analysing topologies and identifying singularities — sometimes called cone points — where a bifurcation of crosses is found. These singularities are characterised by a valence smaller or greater than 4, representing a point in the domain where respectively 3 or 5 quads must be connected to obtain a valid block decomposition of the domain. While previous cross-field based approaches have focused on propagating boundary conditions (BCs) inwards and smoothing the solution, we employ a partial differential equation (PDE) approach based on the Cauchy-Riemann equations. These can be solved using a Laplace equation solver to obtain a smooth solution which can be directly used to accurately locate and analyse singularities and trace separatrices — essentially streamlines — that divide the domain into quad blocks. Crosses are in fact never generated. This pre-cross analysis of the PDE solution reduces the number of extraneous singularities and streamlines forming limit cycles.

Because the solution is smooth, we employ a high-order spectral/*hp* element method to achieve spectral convergence. Because they are calculated from boundary orientation, BCs are discontinuous at corners whose angle is not a multiple of $\pi/2$. Whereas traditional low-order approaches resorted to *ad hoc* smoothing of the BCs at such corners, we use a discontinuous Galerkin (DG) discretisation able to handle BC discontinuities in a discretisation-consistent manner. A DG discretisation further allows us the use of adaptive p-refinement techniques to locally improve the accuracy of the Cauchy-Riemann equations. The result is a highly accurate solution from which a curved block decomposition can be computed and later split into a naturally high-order curvilinear mesh.

These developments are made available on the *Nektar++* spectral/*hp* element open-source framework and its associated set of high-order mesh generation tools, *NekMesh*. This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 675008. This work was also supported by a grant from the Simons Foundation (#426393, David Kopriva).

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