Adaptive Multilevel Stochastic Collocation Method for Elliptic PDEs with Uncertain Data

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ABSTRACT

In this talk, I will present a new adaptive multilevel stochastic collocation method for elliptic PDEs with uncertain data [1]. A hierarchical sequence of adaptive mesh refinements for the spatial approximation is combined with adaptive anisotropic sparse Smolyak grids in the stochastic space in such a way as to minimize computational cost. Adaptive methods have the potential to drastically reduce the number of degrees of freedom and to reach optimal complexity in terms of computing time, even in the case of solutions exhibiting singularities. They usually outperform methods based on uniform mesh refinement, and provide reliable a posteriori error estimators. Coupled with multilevel stochastic algorithms, they can increase the computational efficiency in the sampling process when the stochastic dimension increases and hence provide a general tool to further delay the curse of dimensionality, which is inherent for larger stochastic dimensions.

I provide a rigorous analysis for the convergence and computational complexity of the adaptive multilevel algorithm. Numerical examples demonstrate the reliability of an error control by goal-oriented adaptive methods and the significant decrease in complexity versus uniform spatial refinements, single-level stochastic sampling methods and even multilevel Monte Carlo methods.

REFERENCES

[1] J. Lang, R. Scheichl, Adaptive Multilevel Stochastic Collocation Method for Randomized Elliptic PDEs, Technische Universität Darmstadt, Department of Mathematics, Preprint 2718, 2017.