Theoretical and experimental study of piezoelectric nonlinear resonators for energy harvesting and vibrations control

V. Guillot*, A. Givois†, M. Colin†, O. Thomas† , A. Ture Savadkoohi* ,and C.-H. Lamarque*

* Laboratoire de Tribologie et Dynamique des Systèmes (LTDS), UMR CNRS 5513
Ecole Nationale des Travaux Publics,
3 rue Maurice Audin, 69120 Vaulx-en-Velin, France
Email: vinciane.guillot@entpe.fr, alireza.turesasvadkoohi@entpe.fr, lamarque@entpe.fr

† Laboratoire d'Ingénierie des Systèmes des Systèmes Physiques et Numériques (LISPEN)
Arts et Métiers Paritech, LISPEN EA 7515
8 boulevard Louis XIV, 59000 Lille, France
Email: arthur.givois@ensam.eu, mathieu.r.colin@gmail.com, olivier.thomas@ensam.eu

ABSTRACT

The aim of the work is to control the vibrations or to harvest some vibratory energy from a nonlinear structure through modal energy exchanges of the system. To exchange energy within modes, internal resonances between them are created [1]. We then intend to design the exchange of energy, thanks to the nonlinear terms of the structure, to: (i) perform balance of modal energies, (ii) transfer energy of "dangerous" modes to other one's, (iii) transfer energy to particular modes to harvest it.

A structure composed of a cantilever homogeneous beam patched with one or two piezoelectric materials, at certain positions on the beam, is studied. Two configurations are studied: the first one with a piezoelectric patch on one side on top of the elastic beam, and the second one with two collocated patches. Those two configurations have different effects on the nonlinear terms, because of the axial symmetry or asymmetry it creates. We write the governing nonlinear equations of the system, supposing an Euler Bernoulli beam with large deformations. The nonlinearities of the model come from the large rotation nonlinearity of the beam [2] and the assumption of a nonlinear material behaviour of the piezoelectric patches [3].

The positions of the piezoelectric patches are used to create the intended internal resonance. Written in a general manner, the n^{th} and the m^{th} frequencies (ω_n and ω_m, respectively) are in internal resonance if $\omega_n \approx p\, \omega_m$, with p a natural number and $n \neq m$. Then, the governing equations are expanded on those targeted modes, and studied via a multiple scale method [4]. This study of the equations will allow to identify the nonlinear terms responsible of a possible exchange of energies between the modes.

Some experiments are carried out on two rulers with piezoelectric patches in the two abovementioned configurations (one sided and two-sided piezoelectric patches). The experiments allow us to access some natural frequencies of the structure and their modal shapes. Displacements behaviour of modes in internal resonance are also registered, when excited directly.

Both experiments and the theoretical studies validate that, for some specific positions of piezoelectric patches, a 1:3 and 1:2 internal resonances are created, between the second and third frequency and the third and fourth, respectively. However, only slight energy exchanges between those modes were measured.

Nonlinear reduced order model of the structure are currently under investigation to understand and possibly increase those energy exchanges.

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