On Dynamical Hierarchical Two-Dimensional Models of Multilayer Thermoelastic Piezoelectric Shells

Gia Avalishvili* and Mariam Avalishvili†

* Faculty of Exact and Natural Sciences
I. Javakhishvili Tbilisi State University
3 I. Tchavtchavadze Ave., 0179 Tbilisi, Georgia
e-mail: gavalish@yahoo.com

† School of Science and Technology
University of Georgia, Tbilisi
77a M. Kostava Str., 0175 Tbilisi, Georgia
Email: m.avalishvili@ug.edu.ge

ABSTRACT

Piezoelectric materials are widely used to build engineering smart structures, because of the ease of controlling by voltage, low weight and low power requirements. Inhomogeneous and, in particular, functionally graded materials are used to increase the durability and efficiency of engineering constructions undergoing high mechanical and thermal loads. Various parts of smart devices consisting of piezoelectric materials are often plate or shell like structures, and, therefore, construction and investigation of mathematical models of inhomogeneous thermoelastic piezoelectric plates and shells are important from both theoretical and practical viewpoints.

In this paper, linear dynamical three-dimensional model [1, 2] of thermoelastic piezoelectric shell with variable thickness, which may vanish on a part of the lateral boundary, consisting of several inhomogeneous anisotropic thermoelastic piezoelectric layers with regard to magnetic field is considered. In order to construct dynamical two-dimensional models variational formulation in curvilinear coordinates of the initial-boundary problem corresponding to the three-dimensional model of the shell is considered. Applying spectral approximation method, which is a generalization and extension of the dimensional reduction method suggested by I. Vekua in the classical theory of elasticity for plates with variable thickness [3], a sequence of subspaces with special structure of the spaces corresponding to the original three-dimensional problem is constructed and by projecting the three-dimensional problem on these subspaces a hierarchy of dynamical two-dimensional models is obtained. The constructed two-dimensional initial-boundary value problems are investigated in suitable spaces of vector-valued distributions with respect to the time variable with values in corresponding weighted Sobolev spaces and the existence and uniqueness of solutions are proved. The relationship between the constructed dynamical two-dimensional models and the original three-dimensional one is studied, and it is proved that the sequence of vector-functions of three space variables restored from the solutions of the two-dimensional problems converges in the corresponding function spaces to the exact solution of the three-dimensional initial-boundary value problem and under additional conditions the rate of convergence is estimated.

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