Wave Finite Element method for electromechanical periodic waveguides

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ABSTRACT

A Finite Element method for structures covered with piezoelectric elements was proposed by Thomas et al. [1] who focused on thin piezoelectric patches shunted with independent electrical circuits. The model is based on a condensation of the electrical degrees of freedom in order to recast the system into a standard elastic formulation. However, this method is not applicable when considering interconnections of several patches through an electrical network. In this case, an electromechanical waveguide is created and a wave can thus propagate simultaneously in the mechanical and electrical domains. Because there are electrical nodes that interconnect successive elements, the electrical degrees of freedom cannot be condensed in the mechanical problem. For example, this arises when considering a structure coupled to its electrical analogue for a multimodal vibration damping purpose [2]. Based on a periodic distribution, a Transfer Matrix method can be implemented but it requires the use of external electrical degrees of freedom. A novel Wave Finite Element method dedicated to electromechanical periodic waveguides is then required.

By analogy with displacement and force vectors, electric charge displacement and voltage vectors are defined from the electrical variables along the sides of the unit cell. The equivalent of a dynamic stiffness matrix is obtained from the constitutive equations of the electromechanical problem. The main difference with a purely mechanical formulation [3] or a problem involving independent piezoelectric shunts [4] is that the state vectors include both mechanical and electrical variables [2]. The global “dynamic stiffness matrix” is then rearranged to bring together the left and right variables. With this partitioning, the Wave Finite Element method can be applied and a Transfer Matrix is defined after condensation of the internal mechanical degrees of freedoms. This method offers a convenient numerical model for the analysis of wave propagation in periodic structures involving an interconnected array of piezoelectric patches. The propagation constants can be easily extracted from the Transfer Matrix in order to define the waves propagating through the electromechanical waveguide as well as eventual bandgap effects. In this study, the proposed method is applied to rod and beam models coupled to passive electrical networks made of inductors and transformers. The formulation can then be extended to more complicated one-dimensional structures as long as their mass and stiffness matrices have been defined from a numerical model.

REFERENCES


