Liquid-Gas Phase Transition in Nuclear Matter from a Correlated Approach

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A “hot” day in Barcelona’s history!

A day like today...

19 July 1936
Outline

1. Nuclear Matter at Finite Temperature
2. Self-Consistent Green’s Functions at Finite Temperature
3. Thermodynamical Properties of Nuclear Matter
4. Summary and conclusions
Motivation: “hot” nuclear systems

\[ E \sim 1 \text{ MeV} \Rightarrow T \sim 10^{10} \text{ K} \]

Proto-neutron stars

Chandra X-Ray Observatory

SN 1181 remnant (SNR3C58) and
Pulsar PSRJ0205+6449

AA collisions

Nuclear caloric curve
Motivation: basic considerations

Nuclear Matter

- Infinite system of nucleons
- No surface effects
- Densities $\rho \sim 10^{14} \text{ g cm}^{-3}$
- Model interior of heavy nuclei and neutron stars

Liquid-Gas phase transition

- NN interaction $\Rightarrow$ SR repulsion, LR attraction
- Van der Waals-like EoS
- $T_c \sim E/A|_0 \sim 16 \text{ MeV}$
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Mean-field approach

Symmetric Nuclear Matter EoS - SLy4

- $T=0 \text{ MeV}$
- $T=5 \text{ MeV}$
- $T=10 \text{ MeV}$
- $T=15 \text{ MeV}$
- $T=20 \text{ MeV}$
Motivation: one-body Green’s function

Definition

\[ iG(\vec{r}t, \vec{r}'t') = \left\langle T [\hat{a}(\vec{r}t)\hat{a}^\dagger(\vec{r}'t')] \right\rangle \]

All the one-body properties of a many-body system can be derived from the one-body Green’s function:

\[ \langle \hat{X} \rangle = -i \int d^3r \lim_{\vec{r}' \to \vec{r}, \; t' \to t^+} x(\vec{r})G(\vec{r}t, \vec{r}'t') \]

Two-body properties can also be obtained \((E, S, \ldots)\)
SCGF: Ingredients

- **Main approximation**: decoupling at the level of $G_{III}$
- Includes short-range and tensor correlations
- Full off-shell energy dependence is considered
- Based on the perturbative expansion of the propagator at $T = 0$ and $T \neq 0$
- Thermodynamically consistent (conserving) theory
- **Ladder** includes hole-hole propagation (beyond BHF), which leads to a pairing instability for $T = 0$ ...
- Finite temperature actually solves theoretical problems!
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Ladder approximation

\[ G_{\Pi} = \cdots + \ \text{X terms} + \ \text{H terms} + \ \text{V terms} + \ \cdots \]

- Valid for strong interactions and low densities
- Self-consistency is imposed at each step
- Solved in terms of Dyson’s equation
- Ladder self-energy
- In-medium interaction accounts for ladder scattering
Valid for strong interactions and low densities

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\[ \sum_{\text{SCGF}} = \sum_{\text{part.}} + \sum_{\text{higher order}} + \sum_{\text{medium}} + \sum_{\text{internal}} + \]

- Valid for strong interactions and low densities
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\[ T = + T \]

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Ladder approximation

\[
\langle k_1 k_2 | T(Z_\nu) | k_3 k_4 \rangle = \langle k_1 k_2 | V | k_3 k_4 \rangle \\
+ \mathcal{V} \int \frac{d^3 k_5}{(2\pi)^3} \mathcal{V} \int \frac{d^3 k_6}{(2\pi)^3} \langle k_1 k_2 | V | k_5 k_6 \rangle G^{0}_{II}(Z_\nu ; k_5 k_6) \langle k_5 k_6 | T(Z_\nu) | k_3 k_4 \rangle
\]

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Spectral decomposition of the propagator

- **Momentum-frequency space representation**

\[ G(k, \omega) = \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} A(k, \omega') \left\{ \frac{f(\omega')}{\omega - \omega' - i\eta} + \frac{1 - f(\omega')}{\omega - \omega' + i\eta} \right\} \]

- **Spectral function:**

\[ A(k, \omega) = \frac{-2\text{Im} \Sigma(k, \omega)}{\left[\omega - \frac{k^2}{2m} - \text{Re} \Sigma(k, \omega)\right]^2 + \left[\text{Im} \Sigma(k, \omega)\right]^2} \]
Spectral functions

$A(k, \omega)$ [MeV$^{-1}$] for various temperatures and densities.

- T = 10 MeV
- T = 15 MeV
- T = 20 MeV
- T = 5 MeV

Densities:
- $\rho = 0.10$ fm$^{-3}$
- $\rho = 0.20$ fm$^{-3}$
- $\rho = 0.30$ fm$^{-3}$
- $\rho = 0.40$ fm$^{-3}$
- $\rho = 0.50$ fm$^{-3}$

K-vectors:
- $k = 0$
- $k = k_F$
- $k = 2k_F$
Momentum distributions

\[ n(k) = \langle \hat{a}_k^\dagger \hat{a}_k \rangle = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} A(k, \omega) f(\omega) \]
Thermodynamics of correlated nucleons

Free energy: \( F(\rho, T) = E - TS \)

- Energy (GMK sum rule)

\[
E^{GMK} = \sum_k \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \left\{ \frac{k^2}{2m} + \omega \right\} A(k, \omega)f(\omega)
\]

- Entropy

\[ S = ??? \]

- Can one compute \( S \) from the one-body propagator?
- Does fragmentation affect the TD properties?
Luttinger-Ward formalism

Luttinger and Ward, PR 118,1417 (1960)

- Non-perturbative LW functional for the partition function

\[ \ln Z\{G\} = \tilde{\text{Tr}} \ln \left( -G^{-1} \right) + \tilde{\text{Tr}} \Sigma G - \Phi\{G\} \]

- \(\Phi\)-functional such that:

\[ \frac{\delta \ln Z}{\delta G} \bigg|_{G_0} = 0 \quad \Rightarrow \quad \Sigma\{G\} = \frac{\delta \Phi}{\delta G} \bigg|_{G_0} \]

Baym, PR 127,1391 (1962)

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\[ \Phi \]
\[ \frac{1}{2} \]
\[ \frac{1}{4} \]
\[ \frac{1}{N_f} \]

\[ \Sigma = \frac{\delta \Phi}{\delta G} \]

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- Thermodynamically consistent
Entropy within the LW formalism

\[ S = \left. \frac{\partial T \ln Z}{\partial T} \right|_\mu = S^{DQ} + S' \]

- Dynamical quasi-particle entropy

\[ S^{DQ} = \sum_k \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \sigma(\omega) B(k, \omega) \]

with the statistical factor \( \sigma \) and the \( B \) spectral function:

\[ \sigma(\omega) = -\left\{ f(\omega) \ln[f(\omega)] + [1 - f(\omega)] \ln[1 - f(\omega)] \right\} \]

\[ B(k, \omega) = A(k, \omega) \left[ 1 - \frac{\partial \text{Re} \Sigma(k, \omega)}{\partial \omega} \right] + \frac{\partial \text{Re} G(k, \omega)}{\partial \omega} \Gamma(k, \omega) \]

- Higher order entropy ⇒ neglected at low \( T \)'s

Carneiro and Pethick, PR 11,1106 (1975)

\[ S' = -\frac{\partial}{\partial T} T \Phi\{G\} + \sum_k \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\partial f(\omega)}{\partial T} A(k, \omega) \text{Re} \Sigma(k, \omega) \]

Arnau Rios Huguet (NSCL)
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B spectral function

$\rho = 0.16 \text{ fm}^{-3}, T = 10 \text{ MeV}$

- $B$ has a larger quasi-particle peak
- $B$ has less strength at large energies
- Fragmentation of the qp peak plays a small role
Different approximations

\[ S^{DQ} = \sum_k \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \sigma(\omega) B(k, \omega) \]

\[ S^{QP} = \sum_k \int_{-\infty}^{\infty} d\omega \sigma(\omega) \delta[\omega - \varepsilon_{SCGF}(k)] \]

\[ S^{BHF} = \sum_k \int_{-\infty}^{\infty} d\omega \sigma(\omega) \delta[\omega - \varepsilon_{BHF}(k)] \]

- \( S^{DQ} \sim S^{QP} \Rightarrow \text{width effects unimportant} \)
- \( S^{BHF} \text{ within a } 15\%, S^A \text{ within a } 30\% \)
- \( S^{NK} \text{ too large} \)
- Different lineal slopes \( \Rightarrow \text{different } N(0)'s \)
Thermodynamics of correlated nucleons

Free energy "recipe": \( F = E^{GMK} - TS^{DQ} \)

- Energy (GMK sum rule)
  \[
  E^{GMK} = \sum_k \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{1}{2} \left\{ \frac{k^2}{2m} + \omega \right\} A(k, \omega)f(\omega)
  \]

- Entropy (LW formalism)
  \[
  S^{DQ} = \sum_k \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \sigma(\omega)B(k, \omega)
  \]

- TD consistency
  \[
  \mu = \frac{\partial F}{\partial \rho} \quad \text{vs.} \quad \rho = \nu \int \frac{d^3k}{(2\pi)^3} n(k, \tilde{\mu})
  \]
Thermodynamical Properties of Nuclear Matter

Thermodynamical consistency

- SCGF + LW yields $\mu \sim \tilde{\mu}$
- BHF violates HvH theorem by 20 MeV
- Far from correct saturation

$$\mu = \frac{\partial F}{\partial \rho} \iff \rho = \nu \int \frac{d^3k}{(2\pi)^3} n(k, \tilde{\mu})$$
Liquid-gas phase transition

\[ p = \rho (\tilde{\mu} - F/A) \]

- Spinodal zone related to mechanical instability
- Maxwell construction sets phase coexistence
Liquid-gas phase transition

- $T_{c}^{BHF} \gg T_{c}^{SCGF}$
- Very different critical behaviour!
- Upper estimate of finite nuclei $T_{c}$
Summary

- The SCGF scheme is a consistent framework for nuclear many-body calculations at finite temperatures.
- The LW formalism can be used to find the TD properties of a many-body system from the one-body propagator.
- First time that the correlated entropy is computed for nuclear matter.
- Different realistic approaches lead to different $T_c \Rightarrow$ room for improvement!
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Outlook

- Dependence on the 2-body NN potential
- Inclusion of 3-body effects
- Different methods to obtain the TD properties of the system
- $\rho$ and $T$ dependences of the microscopic properties
- Isospin asymmetry and its consequences
- Pairing phase transition beyond quasi-particle approach
- Extension to time-dependent systems (HIC)
Thank you!
For further reading I

T. Frick and H. Müther,  
*Self-consistent solution to the nuclear many-body problem at finite temperature*,  

T. Frick, H. Müther, A. Rios, A. Polls and A. Ramos,  
*Correlations in hot asymmetric nuclear matter*,  

A. Rios, A. Polls and H. Müther,  
*Sum rules of single-particle spectral functions in hot asymmetric nuclear matter*,  

Realistic NN interactions

NN interaction properties

- NN scattering phase-shifts
- Deuteron phenomenology
  - Bound state
  - Tensor component
- Different phase-shift equivalent potentials
  CDBONN, Av18, etc.
Analytical continuation

- Spectral decomposition of Matsubara coefficients

\[ G(k, z_\nu) = \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{A(k, \omega')}{z_\nu - \omega'} \]

- Analytical continuation

\[ G(k, z_\nu) \rightarrow G(k, z) \]

- Can be done under certain assumptions


- Relation to the retarded propagator

\[ G(k, z) = \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{A(k, \omega')}{z - \omega'} \]

\[ \rightarrow \omega + i\eta \]

\[ G^R(k, \omega) = \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{A(k, \omega')}{\omega - \omega' + i\eta} \]
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Depletion

- $T$ dependence $\Rightarrow f(\omega)$
- $\rho$ dependence $\Rightarrow$ correlations
- Measure of both thermal and dynamical correlations
Correlated and non-correlated $n(k)$

- Less populated at low $k$
- More populated at high $k$
- Strong fall-off near $k_F$

$r = 0.32$ fm$^{-3}$, $T = 5$ MeV
**B spectral function**

- Different $\rho$ and $T$ dependence
- High energy tails measure importance of correlations
Mean-field to correlated energy ratios

Kinetic energy $\Rightarrow \rho$ and $T$ independent
Potential energy $\Rightarrow$ large modification
Free energy and $\tilde{\mu}$

$F/A$ minimum disappears with $T \Rightarrow T_{fl}$

$T_c$ where $F/A$ looses inflexion point

$\mu$ and $\tilde{\mu}$ coincide within 2 MeV