Many-Body challenges in nuclear astrophysics

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Recent Progress in Many-Body Theories

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Outline

1. Introduction

2. Reactions involving light nuclei
   - Extending ab initio approaches to reactions
   - Fermion Molecular Dynamics

3. Weak processes in supernovae
   - Electron capture
   - Neutrino-nucleus reactions

4. Nucleosynthesis heavy elements (r-process)

5. Conclusions
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Nuclear Astrophysics

Improved observational capabilities

- New radioactive ion beam facilities (RIBF, SPIRAL 2, FAIR) are being built or developed that allow to explore many nuclei produced in explosive events.
- We need improved many body models to fully exploit the potential offered by these facilities.
Nuclear Interaction and Degrees of Freedom
Green’s Function Monte Carlo

parameters of three-body force Illinois-2 fitted to reproduce spectra

Wiringa, Pieper, PRL 89 (2002) 182501
No-Core Shell Model

- Solves the nuclear many body problem for light nuclei using realistic NN forces.
- The calculations are done in a large but finite harmonic-oscillator basis using an effective interaction derived via an unitary Lee-Suzuki transformation.

Problems describing states with a clear cluster structure like the Hoyle state ($0^+$ at 7.654 MeV in $^{12}$C)

Quest for a unified description of nuclei

- Free NN interaction
- Effective interaction derived from free interaction in oscillator basis
- Effective interaction fitted to known spectra
- Energy functional fitted to binding energies and radii

Limits of nuclear existence

Ab initio few-body calculations
No-Core Shell Model
G-matrix

Density Functional Theory
Selfconsistent Mean Field
Ab initio approaches

Several attempts have been pursued to extend *ab initio* approaches to astrophysical reactions.

- **Variational Monte Carlo calculations** have been done for $^2\text{H}(\alpha, \gamma)^6\text{Li}$, $^3\text{H}(\alpha, \gamma)^7\text{Li}$, and $^3\text{He}(\alpha, \gamma)^7\text{Be}$ capture reactions (Nollet et al, PRC 63 (2001) 054002).
- $^7\text{Be}(p, \gamma)^8\text{B}$ have been computed using the no-core shell model approach (Navrátil et al 73 (2006) 065801).
- They are not yet completely *ab initio* as the scattering states are computed using simple potential models.

$$S(E) = \sigma(E)E e^{2\pi\eta(E)}$$

We need a consistent description of bound, resonant and scattering states.
Fermion Molecular Dynamics

**Fermionic**

\[ | Q \rangle = \mathcal{A} (| q_1 \rangle \otimes \cdots \otimes | q_A \rangle) \]

- antisymmetrized $A$-body state

**Molecular**

single particle states

\[ \langle \vec{x} | q \rangle = \sum_i c_i \exp \left\{ - \frac{(\vec{x} - \vec{b}_i)^2}{2a_i} \right\} \otimes | \chi_{i}^{\uparrow}, \chi_{i}^{\downarrow} \rangle \otimes | \xi \rangle \]

- Gaussian wave-packets in phase-space (complex parameter $\vec{b}_i$ encodes mean position and mean momentum), spin is free, isospin is fixed.
- width $a_i$ (and $\vec{b}_i$ ) are independent variational parameters for each wave packet.

**Dynamics**

- Interaction derived from Argonne V18 interaction by explicit inclusion of short-range central and tensor correlations via the Unitary Correlation Operator Method (Neff & Feldmeier, NPA 713 (2003) 311)
- Projection on parity, linear and angular momentum (before or after variation).
Hoyle State and triple alpha reaction
12C description

\[
\begin{array}{c}
\begin{array}{cccc}
12C & FMD \alpha/\text{clusterExperiment} & 12C
\end{array}
\end{array}
\]

\[
\begin{array}{c}
\begin{array}{cccc}
12C & FMD \alpha/\text{clusterExperiment} & 12C
\end{array}
\end{array}
\]

![Diagram of 12C levels and quasi-elastic scattering](image)
Important Configurations and Occupation numbers

- Most important contributions to Hoyle state and ground state (FMD states not orthogonal)

\[
\begin{align*}
\langle \cdot | 0^+_1 \rangle &= 0.30 & \langle \cdot | 0^+_1 \rangle &= 0.25 & \langle \cdot | 0^+_1 \rangle &= 0.15 & \langle \cdot | 0^+_1 \rangle &= 0.08 & \langle \cdot | 0^+_1 \rangle &= 0.94 \\
\langle \cdot | 0^+_2 \rangle &= 0.72 & \langle \cdot | 0^+_2 \rangle &= 0.71 & \langle \cdot | 0^+_2 \rangle &= 0.61 & \langle \cdot | 0^+_2 \rangle &= 0.61 & \langle \cdot | 0^+_2 \rangle &= 0.04
\end{align*}
\]

- Harmonic Oscillator Occupation Numbers.
Astrophysical applications

Fusion cross-sections for Oxygen isotopes:
(Neff et al, nucl-th/0703030)

Radiative alpha capture:
(Neff et al, preliminary)
Electron capture during the collapse

Important processes:

- Neutrino transport (Boltzmann equation):
  \[ \nu + A \rightleftharpoons \nu + A \] (trapping)
  \[ \nu + e^- \rightleftharpoons \nu + e^- \] (thermalization)
  cross sections \( \sim E_{\nu}^2 \)

- Electron capture on protons:
  \[ e^- + p \rightleftharpoons n + \nu_e \]

- Electron capture on nuclei:
  \[ e^- + A(Z, N) \rightleftharpoons A(Z-1, N+1) + \nu_e \]

Traditional treatment (Independent particle model)
suppresses electron capture on nuclei for \( N = 40 \).

- Gamow-Teller strength can be determined by charge exchange reactions

- Theory is needed to account for finite temperature effects (excited states)
  (Shell-Model diagonalizations, Shell-Model (Auxiliary Field) Monte Carlo
Level densities with Auxiliary Field Monte Carlo

- Level densities are necessary for the calculation of cross sections in medium and heavy nuclei using the Hauser-Feshbach statistical model and for the calculation of partition functions.
- AFMC has been extensively been used for the calculation of level densities using schematical interactions (pairing plus quadrupole) due to sign problem (Alhassid, Langanke, …).
- Very recently has been possible to avoid the sign problem shifting the MC integration to the Hartree minimum of the fields.
- This allows to use fully realistic interactions.

D. Dean, INPC 2007 talk
Laboratory vs. stellar electron capture

Capture of K-shell electrons to tail of GT strength distribution. Parent nucleus in the ground state.

Capture of electrons from the high energy tail of the FD distribution. Capture to states with large GT matrix elements (GT resonance). Thermal ensemble of initial states.
GT strength in $^{48}\text{Sc}$, $^{50}\text{V}$, $^{58}\text{Ni}$, $^{64}\text{Ni}$ also measured.
Electron capture on nuclei dominates over capture on protons

All models converge to a “norm” stellar core at the moment of shock formation.
Neutrino interactions during the collapse

Bruenn and Haxton (1991)
Based on results for $^{56}\text{Fe}$

- **Elastic scattering:**
  \[ \nu + A \rightleftharpoons \nu + A \] (trapping)

- **Absorption:**
  \[ \nu_e + (N, Z) \rightleftharpoons e^- + (N - 1, Z + 1) \]

- **$\nu$-$e$ scattering:**
  \[ \nu + e^- \rightleftharpoons \nu + e^- \] (thermalization)

- **Inelastic $\nu$-nuclei scattering:**
  \[ \nu + A \rightleftharpoons \nu + A^* \]

Inelastic Neutrino-nucleus interactions had not been included in collapse simulations.
Neutrino scattering from $(e, e')$

\[
T(GT_0) \sim \sum_{i} t_z(i) \bar{S}_i
\]

\[
T(M1) = \left\{ \frac{1}{2} (\bar{L}_p - \bar{L}_n) + (g_p - g_n) \sum_{i} t_z(i) \bar{S}_i \right\} \mu_N
\]

*M1 data give $GT_0$ information if Orbital contribution can be removed*
Neutrino Scattering from \((e, e')\)


\[ B(M1) (\mu N^2) \]

\[ 52\text{Cr} \]

\[ \sigma (10^{-42} \text{ cm}^2) \]

\[ M1 \text{ expt.} \]

\[ GT_0 \text{ SM} \]

\[ M1 \text{ corrected} \]

\[ 50\text{Ti} \]

\[ 52\text{Cr} \]

\[ 54\text{Fe} \]

\[ \text{M1 data (S-Dalinac) can be used to constrain supernovae inelastic neutrino cross sections.} \]
Influence on neutrino spectra

- A future detection of a close by supernova could bring information about supernova dynamics.
- We have done detailed simulations and shown that the spectrum of the initial $\nu_e$ burst is affected by the inclusion of inelastic neutrino scattering with nuclei (B. Müller et al).
- At later times (relevant for nucleosynthesis) spectra is unchanged as all nuclei are dissociated.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\langle \sigma \rangle \times 10^{-42}$ cm$^2$</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e$</td>
<td>0.106</td>
<td>0.110</td>
</tr>
<tr>
<td>$d$</td>
<td>4.92</td>
<td>5.36</td>
</tr>
<tr>
<td>$^{12}$C</td>
<td>0.050</td>
<td>0.080</td>
</tr>
<tr>
<td>$^{16}$O</td>
<td>0.0053</td>
<td>0.0128</td>
</tr>
<tr>
<td>$^{40}$Ar</td>
<td>13.4</td>
<td>15.1</td>
</tr>
<tr>
<td>$^{56}$Fe</td>
<td>6.2</td>
<td>7.5</td>
</tr>
<tr>
<td>$^{208}$Pb</td>
<td>103.3</td>
<td>124.5</td>
</tr>
</tbody>
</table>
The r-process is responsible for the synthesis of half the nuclei with $A > 60$ including U, Th and maybe the super-heavies.

\[ T \approx 100 \text{ keV} \quad n \gtrsim 10^{20} \text{ cm}^{-3} \text{ implies } \tau_n \ll \tau_\beta \]

\[(n, \gamma) \rightleftharpoons (\gamma, n) \text{ implies } S_n \approx 2–3 \text{ MeV}\]
Neutrino-driven wind scenario

- Neutrino-wind from (cooling) NS
  \[ \nu_e + n \rightarrow e^- + p \]
  \[ \bar{\nu}_e + p \rightarrow e^+ + n \]
- \( \alpha \)-process (formation seed nuclei)
  \[ ^4\text{He}(\alpha n, \gamma) ^9\text{Be}(\alpha, n) ^{12}\text{C} \]
  \[ ^4\text{He}(t, \gamma) ^7\text{Li}(n, \gamma) ^8\text{Li}(\alpha, n) ^{11}\text{Li} \]

Main parameter determining the nucleosynthesis is the neutron to seed ratio (\( \sim 100 \))
r-process needs

- Fission rates and distributions:
  - n-induced
  - spontaneous
  - \(\beta\)-delayed

- \(\beta\)-delayed n-emission branchings (final abundances)

- \(\beta\)-decay half-lives (abundance and process speed)

- Seed production rates (\(\alpha\alpha, \alpha n, \alpha 2n, ..\))

- n-capture rates
  - for \(A>130\) in slow freezeout
  - for \(A<130\) maybe in a “weak” r-process?

- Masses (Sn) (location of the path)

- \(\nu\)-physics?

Figure from H. Schatz
Masses for r-process nuclei

We need reliable models to determine masses for r-process nuclei (DFT with functional connected to QCD via EFT)

![Diagram showing two-neutron separation energies and mass formulae](image)
The $N=126$ nuclei are not yet accessible experimentally. However, in a recent experiment at the FRS (GSI) several nuclei were produced approaching the $N = 126$ (Kurtukian-Nieto et al, 2007)
Influence on the input data

Two different sets of neutron capture rates, half-lives, fission-barriers, …
Nuclear astrophysics requires the knowledge of the relevant nuclear physics input combined with state-of-the-art hydrodynamic modelling, which then leads to testable predictions with actual observations.

Future challenges for nuclear structure theory are the extension of ab-initio calculations based on realistic nucleon-nucleon interactions to nuclei beyond $A = 12$, and extend ab-initio models to the description of nuclear reactions.

The r-process requires large amounts of data. We need to develop models that are generally applicable and at the same time describe reliable the properties of the involved nuclei.