

Proton correlations as a function of nucleon asymmetry

- Green's function method/framework
- Illustrations: atoms, liquid ^3He & nuclei
- Correlations for stable closed-shell nuclei
- Exciting physics for $N \gg Z$ or $Z \gg N$ nuclei?
 - Critical quantity $\delta=(N-Z)/A \Rightarrow$ asymmetry
 - What to look for? Motivation
 - Solid framework for extrapolation
 - Some recent results and developments
 - Outlook

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Green's function ingredients

Single-particle propagator (Green's function):

$$G(\alpha, \beta; E) = \sum_m \frac{\langle \Psi_0^A | a_\alpha | \Psi_m^{A+1} \rangle \langle \Psi_m^{A+1} | a_\beta^\dagger | \Psi_0^A \rangle}{E - (E_m^{A+1} - E_0^A) + i\eta} \quad \leftarrow \text{Particle part}$$

$$+ \sum_n \frac{\langle \Psi_0^A | a_\beta^\dagger | \Psi_n^{A-1} \rangle \langle \Psi_n^{A-1} | a_\alpha | \Psi_0^A \rangle}{E - (E_0^A - E_n^{A-1}) - i\eta} \quad \leftarrow \text{Hole part}$$

Spectral functions: $S_h(\alpha; E) = \sum_n \left| \langle \Psi_n^{A-1} | a_\alpha | \Psi_0^A \rangle \right|^2 \delta(E - (E_0^A - E_n^{A-1}))$

Spectroscopic factor: $S_{\ell j}^n = \int dr r^2 \left| \langle \Psi_n^{A-1} | a_{r\ell j} | \Psi_0^A \rangle \right|^2$

Occupation numbers: $n(\alpha) = \int_{-\infty}^{\varepsilon_F^-} S_h(\alpha; E) dE = \langle \Psi_0^A | a_\alpha^\dagger a_\alpha | \Psi_0^A \rangle$

Below $\varepsilon_F^- = E_0^A - E_0^{A-1} \quad \Rightarrow \quad S_h(\alpha; E) = \frac{1}{\pi} \text{Im} G(\alpha, \alpha; E)$

Density matrix; natural orbits; Galitskii-Migdal energy sum rule ...

Theory & Framework

Theory
hard...



- Calculations: Include SRC and LRC as good as possible
- Compare with experiment; add more physics

Framework
"easy"

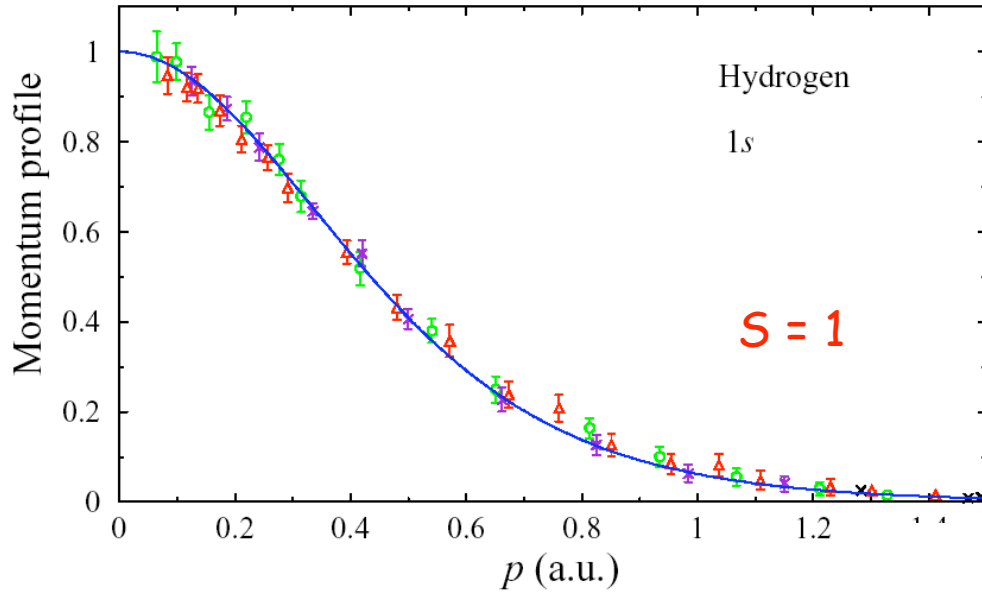


- Determine propagator from data!?!
- $(e,2e)$ & $(e,e'p)$ reaction below the Fermi energy
- Elastic scattering data above the Fermi energy

Answers for example:

What do nucleons do in the nucleus
and how does their behavior change
as a function of asymmetry

Atoms

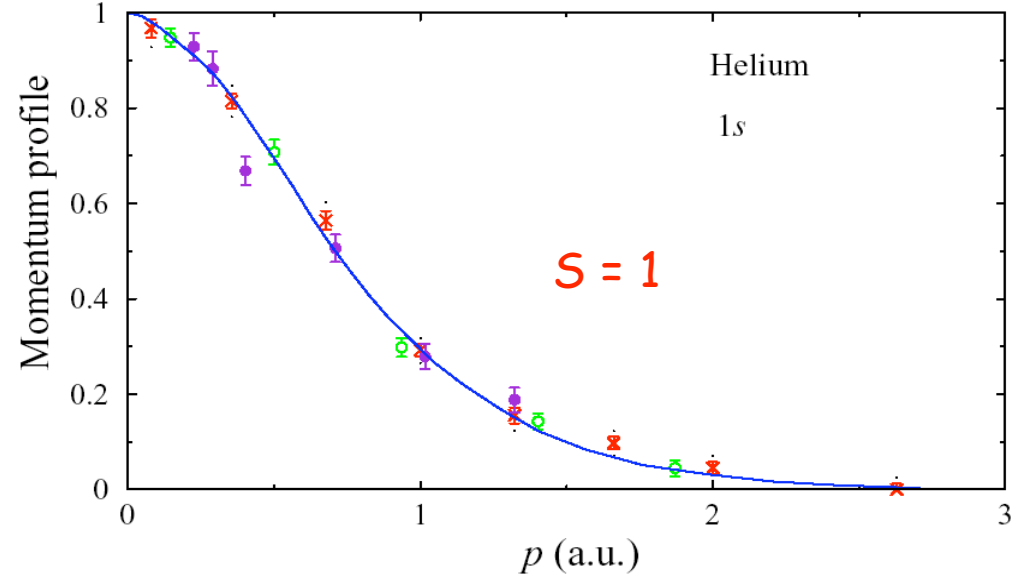


$$\varphi_{1s}(q) = 2^{3/2} \pi \frac{1}{(1+q^2)^2}$$

Hydrogen 1s wave function
"seen" experimentally
Phys. Lett. 86A, 139 (1981)

And so on for other atoms ...

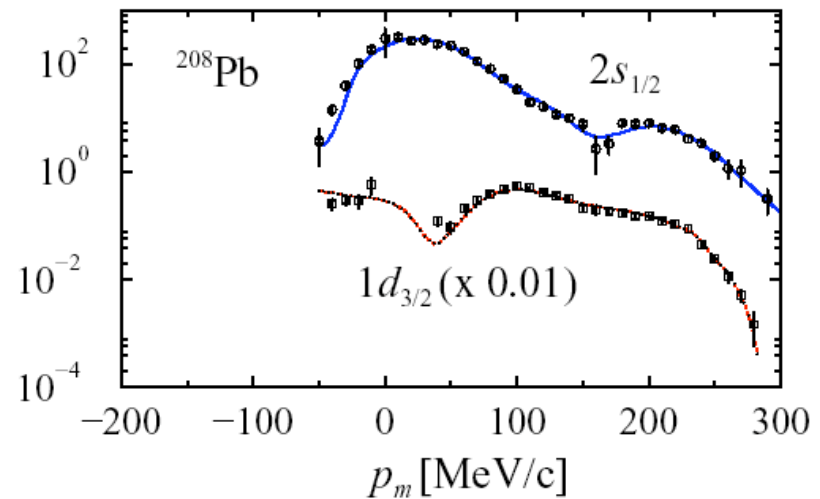
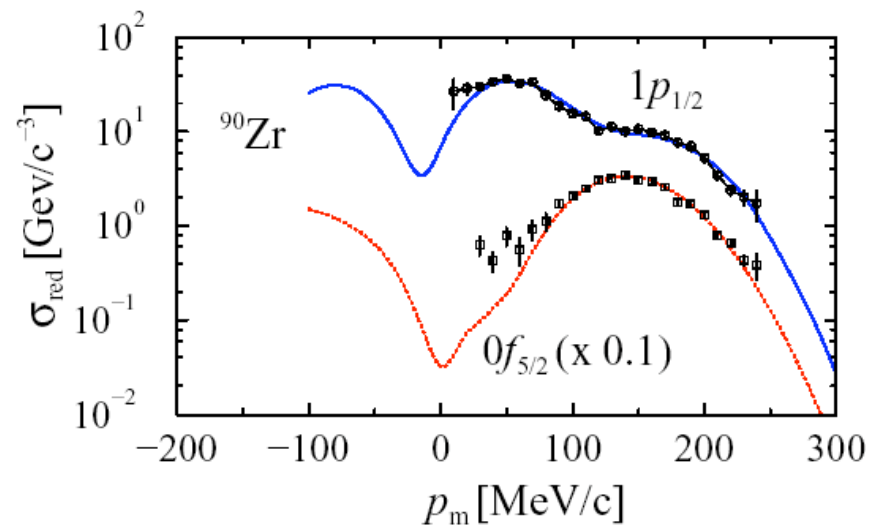
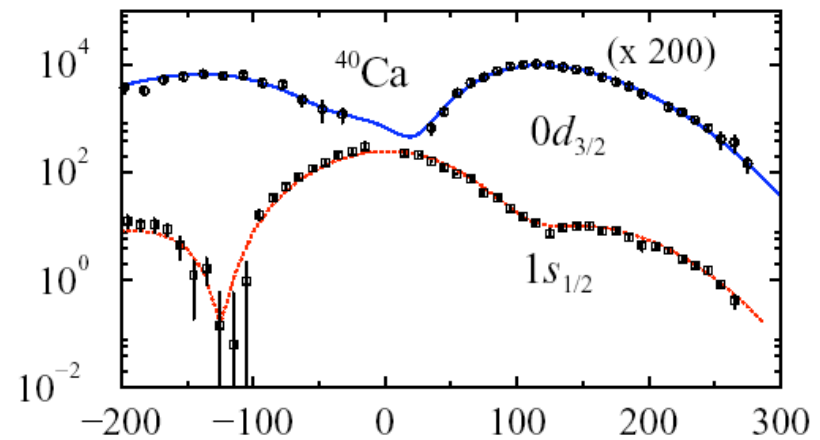
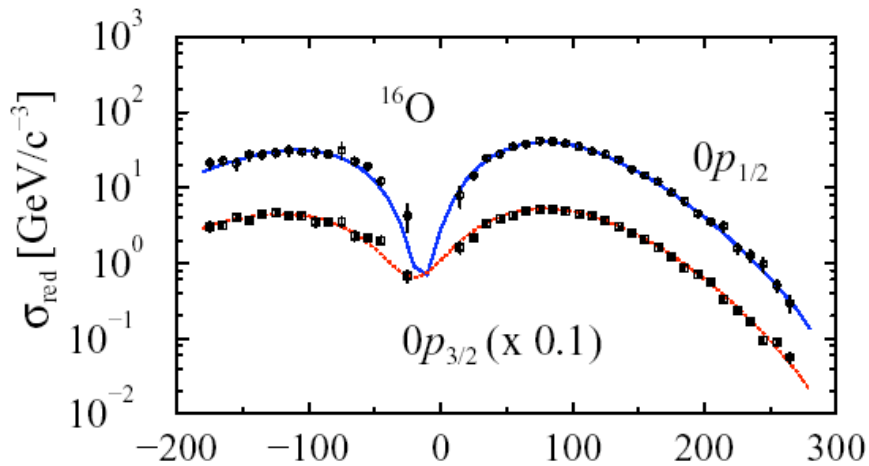
Helium
in Phys. Rev. A8, 2494 (1973)



Closed-shell atoms: $n(\alpha) = 0$ or 1

Nuclei (e,e'p) reaction

NIKHEF data, L. Lapikás, Nucl. Phys. A553, 297c (1993)



Wave functions as expected, except

Nucleon propagator and dripline physics 5

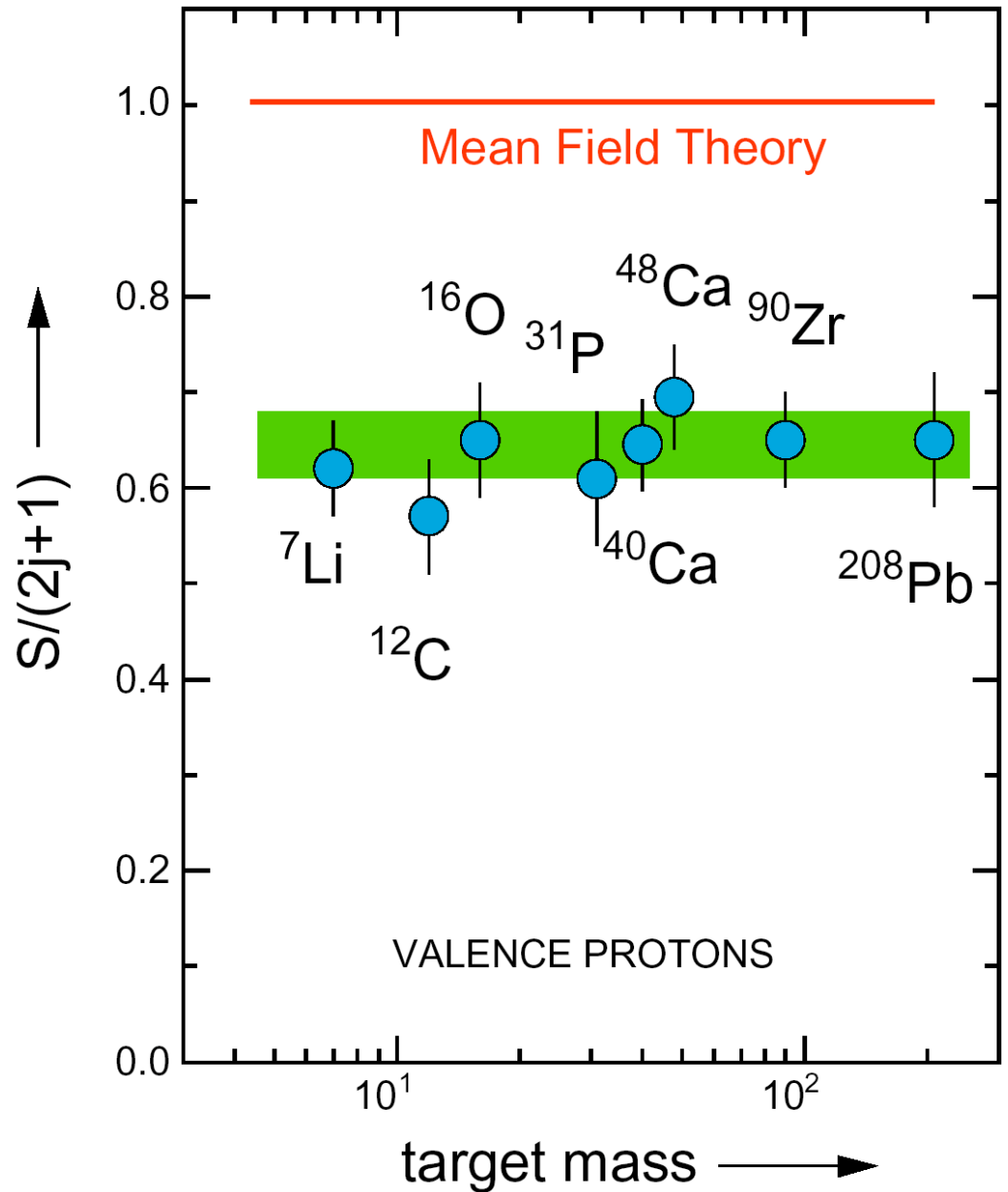
Removal probability for
valence protons
from
NIKHEF data

L. Lapikás, Nucl. Phys. A553,297c (1993)

$S \approx 0.65$ for valence protons
Reduction \Rightarrow both SRC and LRC

Note:

We have seen mostly
data for removal of
valence protons



Two effects associated with short-range correlations

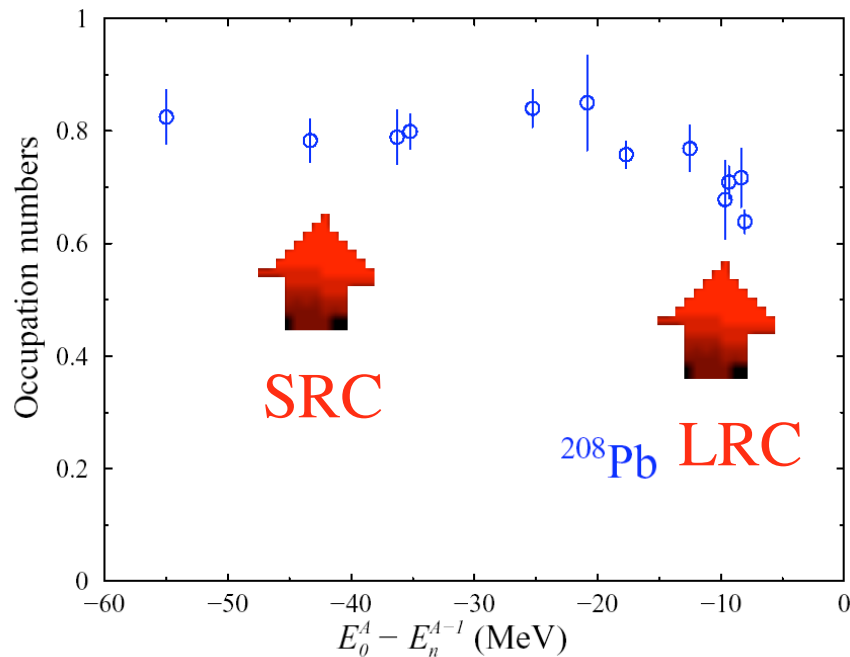
- Depletion of the Fermi sea
- Admixture of high-momentum components

Recent data confirm both aspects (predicted by nuclear matter results)

M. van Batenburg & L. Lapikás from $^{208}\text{Pb} (e, e' p) ^{207}\text{Tl}$

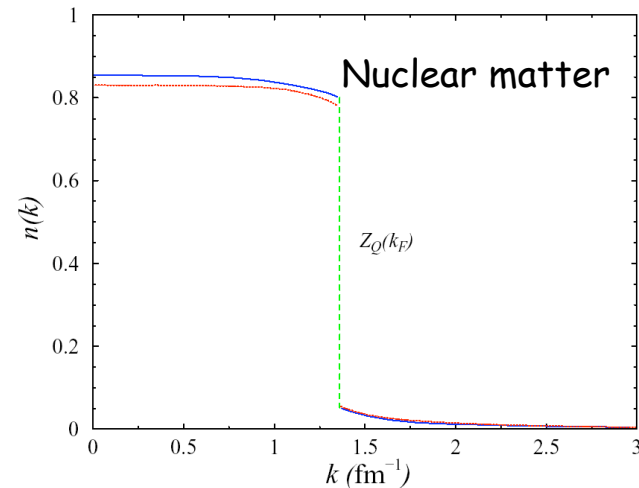
NIKHEF in preparation

Occupation of deeply-bound proton levels from EXPERIMENT



Up to 100 MeV missing energy and
270 MeV/c missing momentum

Covers the whole mean-field domain
for the FIRST time!!

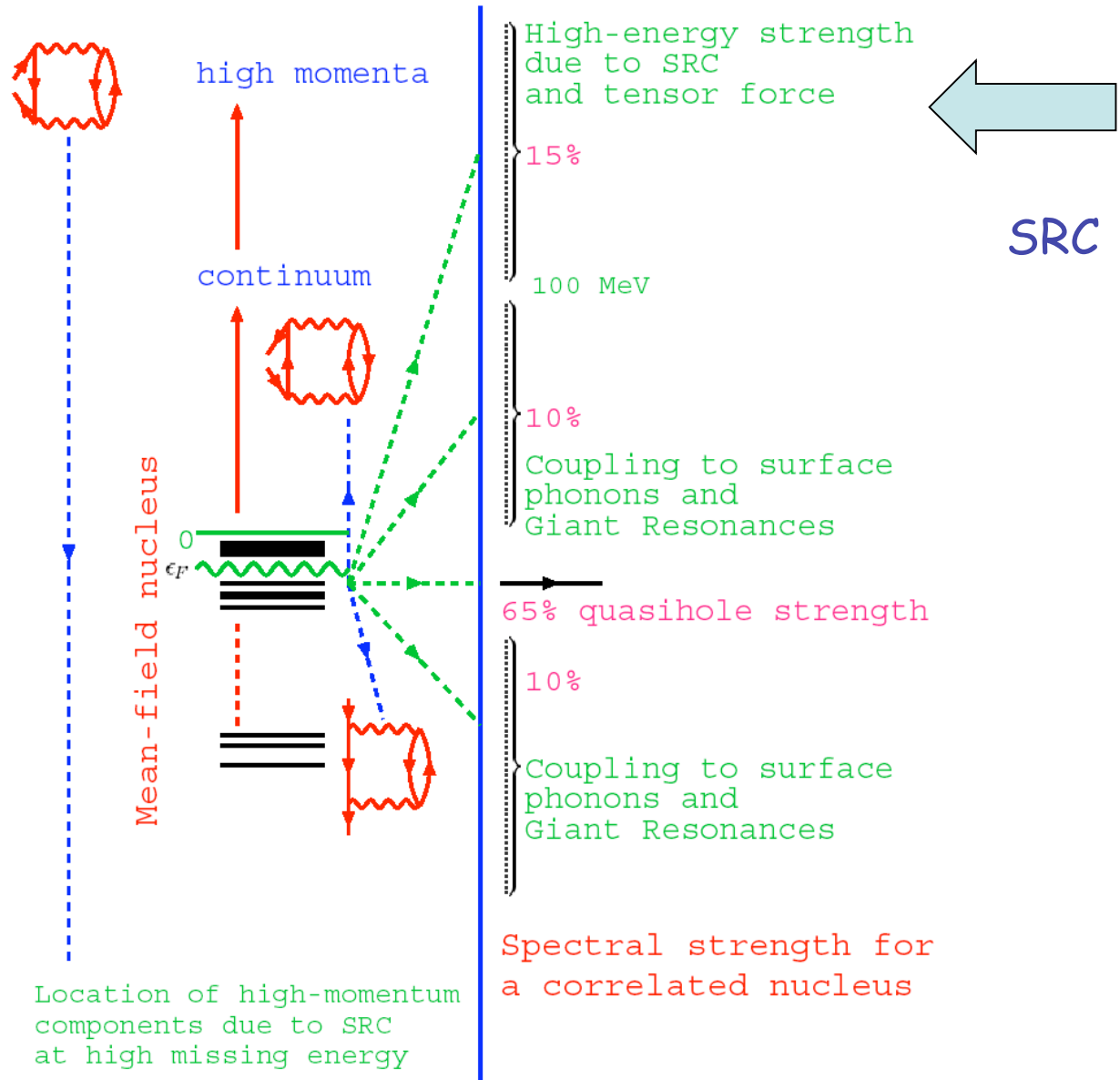
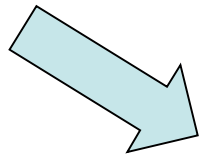


Confirms predictions for depletion

$n(0) \Rightarrow$ 0.85 Reid
0.87 Argonne V18
0.89 CDBonn

Location of single-particle strength in nuclei

SRC



Correlations for nuclei with N very different from Z ?

⇒ Radioactive beam facilities

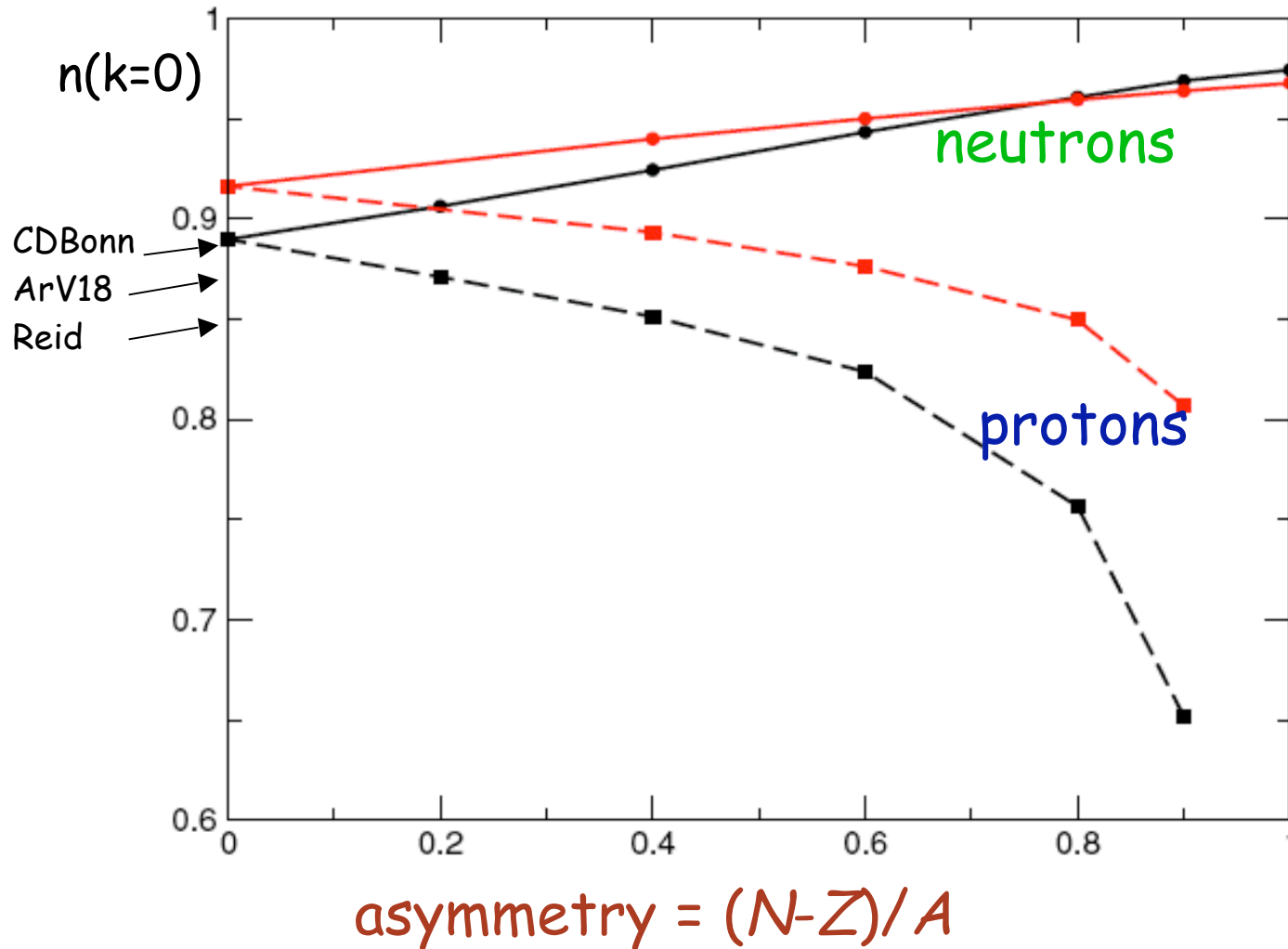
Nuclei are TWO-component Fermi liquids

- SRC about the same between pp, np, and nn
- Tensor force disappears for n when $N \gg Z$ but ...
- Any surprises?
- Ideally: quantitative predictions based on solid foundation

Some pointers: both from theory and experiment

SCGF for isospin-polarized nuclear matter including SRC \Rightarrow momentum distribution

Frick *et al.*
 PRC71,014313(2005)



0.16 fm^{-3}

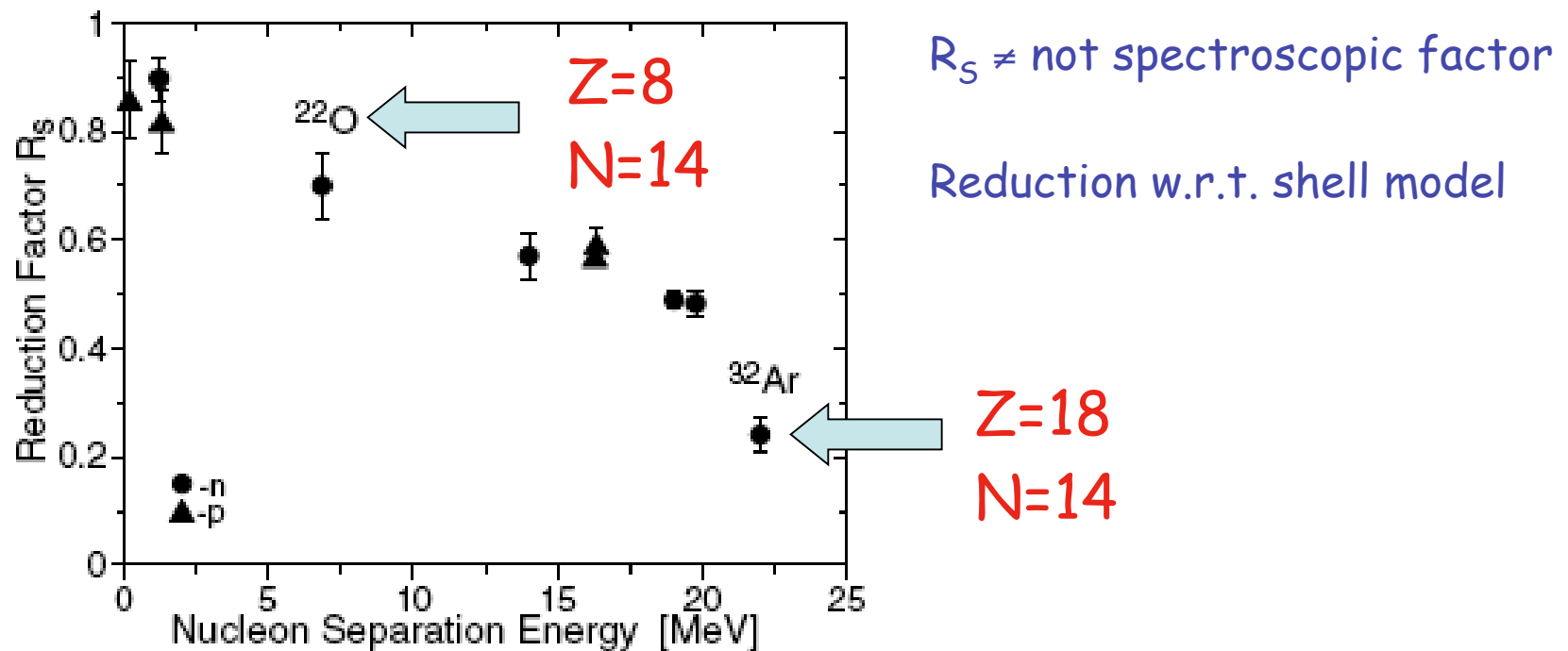
0.32 fm^{-3}

SRC
 can be handled

A. Gade et al., Phys. Rev. Lett. 93, 042501 (2004)

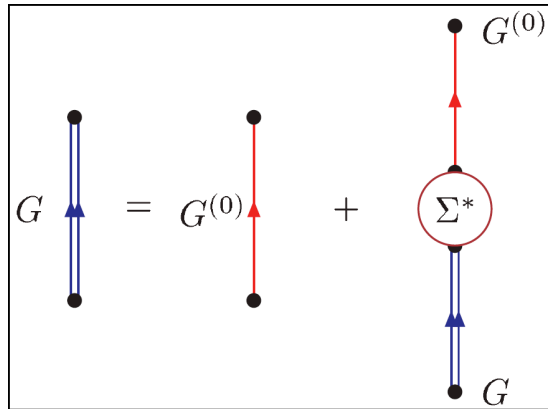
Program at MSU initiated by Gregers Hansen

P. G. Hansen and J. A. Tostevin, Annu. Rev. Nucl. Part. Sci. **53**, 219 (2003)



neutrons more correlated with increasing proton number and accompanying increasing separation energy.

Dyson Equation and "experiment"



Equivalent to ...

Schrödinger-like equation with: $E_n^- = E_0^N - E_n^{N-1}$

Self-energy: non-local, energy-dependent potential

With energy dependence: spectroscopic factors < 1

\Rightarrow as observed in (e,e'p)

$$-\frac{\hbar^2 \nabla^2}{2m} \langle \Psi_n^{N-1} | a_{\vec{r}m} | \Psi_0^N \rangle + \sum_{m'} \int d\vec{r}' \Sigma^*(\vec{r}m, \vec{r}'m'; E_n^-) \langle \Psi_n^{N-1} | a_{\vec{r}'m'} | \Psi_0^N \rangle = E_n^- \langle \Psi_n^{N-1} | a_{\vec{r}m} | \Psi_0^N \rangle$$

$$S = \left| \langle \Psi_n^{N-1} | a_{\alpha_{qh}} | \Psi_0^N \rangle \right|^2 = \frac{1}{1 - \left. \frac{\partial \Sigma^*(\alpha_{qh}, \alpha_{qh}; E)}{\partial E} \right|_{E_n^-}}$$

α_{qh} solution of DE at E_n^-

DE yields

$$\langle \Psi_n^{N-1} | a_{\vec{r}m} | \Psi_0^N \rangle = \psi_n^{N-1}(\vec{r}m)$$

$$\langle \Psi_0^N | a_{\vec{r}m} | \Psi_k^{N+1} \rangle = \psi_k^{N+1}(\vec{r}m)$$

$$\langle \Psi_E^{c,N-1} | a_{\vec{r}m} | \Psi_0^N \rangle = \chi_c^{N-1}(\vec{r}m; E)$$

$$\langle \Psi_0^N | a_{\vec{r}m} | \Psi_E^{c,N+1} \rangle = \chi_c^{N+1}(\vec{r}m; E)$$

Bound states in N-1

Bound states in N+1

Scattering states in N-1

Elastic scattering in N+1

Elastic scattering wave function for (p,p) or (n,n)

FRAMEWORK FOR EXTRAPOLATIONS BASED ON EXPERIMENTAL DATA

“Mahaux analysis” \Rightarrow Dispersive Optical Model (DOM)

C. Mahaux and R. Sartor, *Adv. Nucl. Phys.* **20**, 1 (1991)

There is empirical information about the nucleon self-energy!!

\Rightarrow Optical potential to analyze elastic nucleon scattering data

\Rightarrow Extend analysis from $A+1$ to include structure information in $A-1 \Rightarrow (e,e'p)$ data

\Rightarrow Employ dispersion relation between real and imaginary part of self-energy

Recent extension

Combined analysis of protons in ^{40}Ca and ^{48}Ca

Charity, Sobotka, & WD nucl-ex/0605026, *Phys. Rev. Lett.* **97**, 162503 (2006)

Large energy window (> 200 MeV)

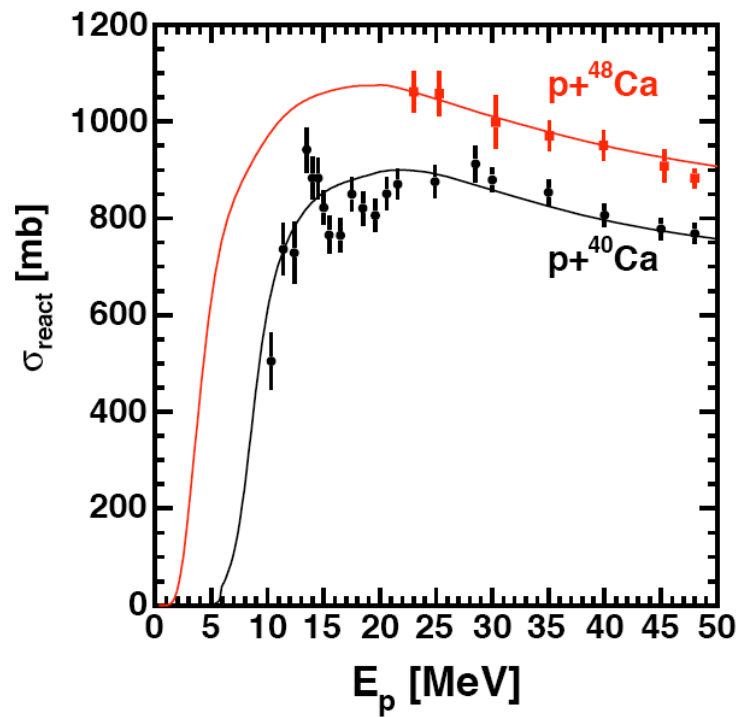
Goal: Extract asymmetry dependence $\Rightarrow \delta = (N - Z)/A$

\Rightarrow **Predict** proton properties at large asymmetry $\Rightarrow ^{60}\text{Ca}$

\Rightarrow **Predict** neutron properties ... the dripline

based on data!

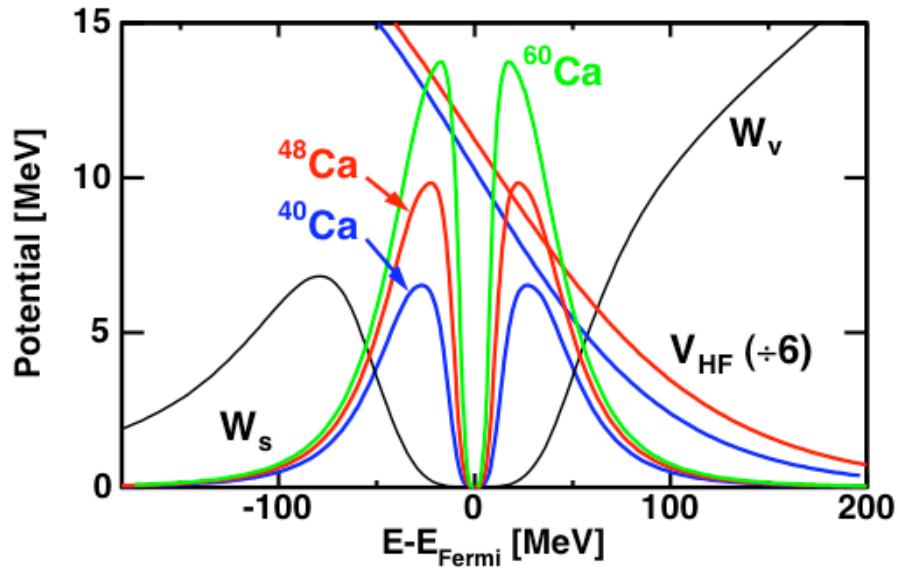
Reaction cross section ^{40}Ca and ^{48}Ca



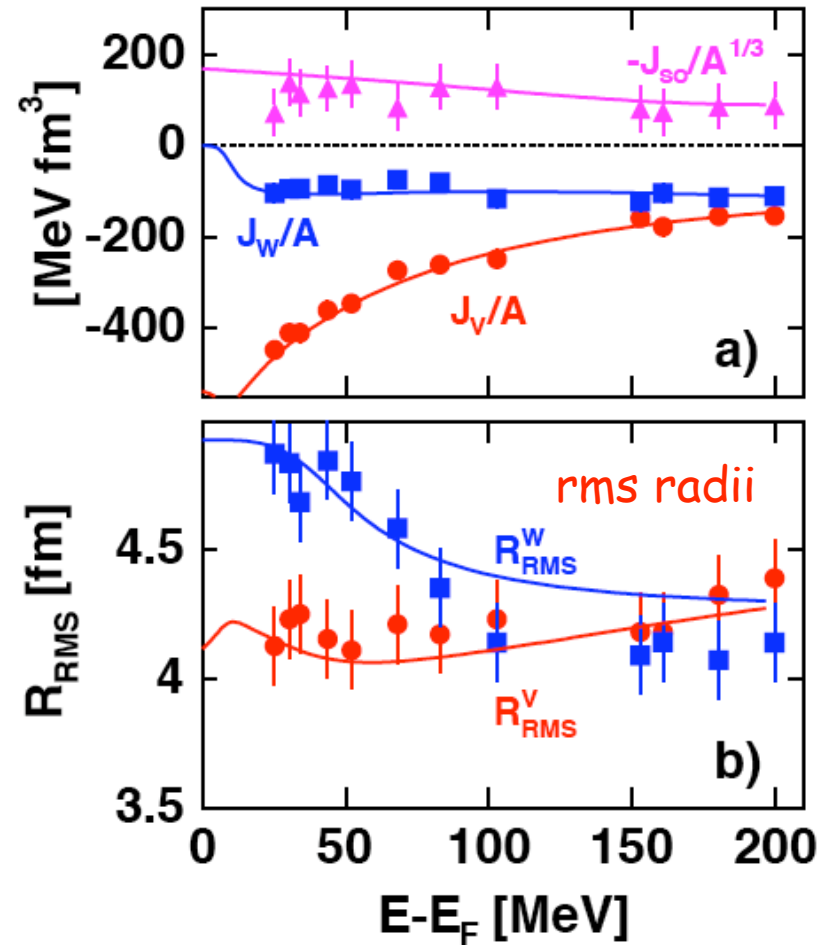
Loss of flux in the elastic channel

Potentials

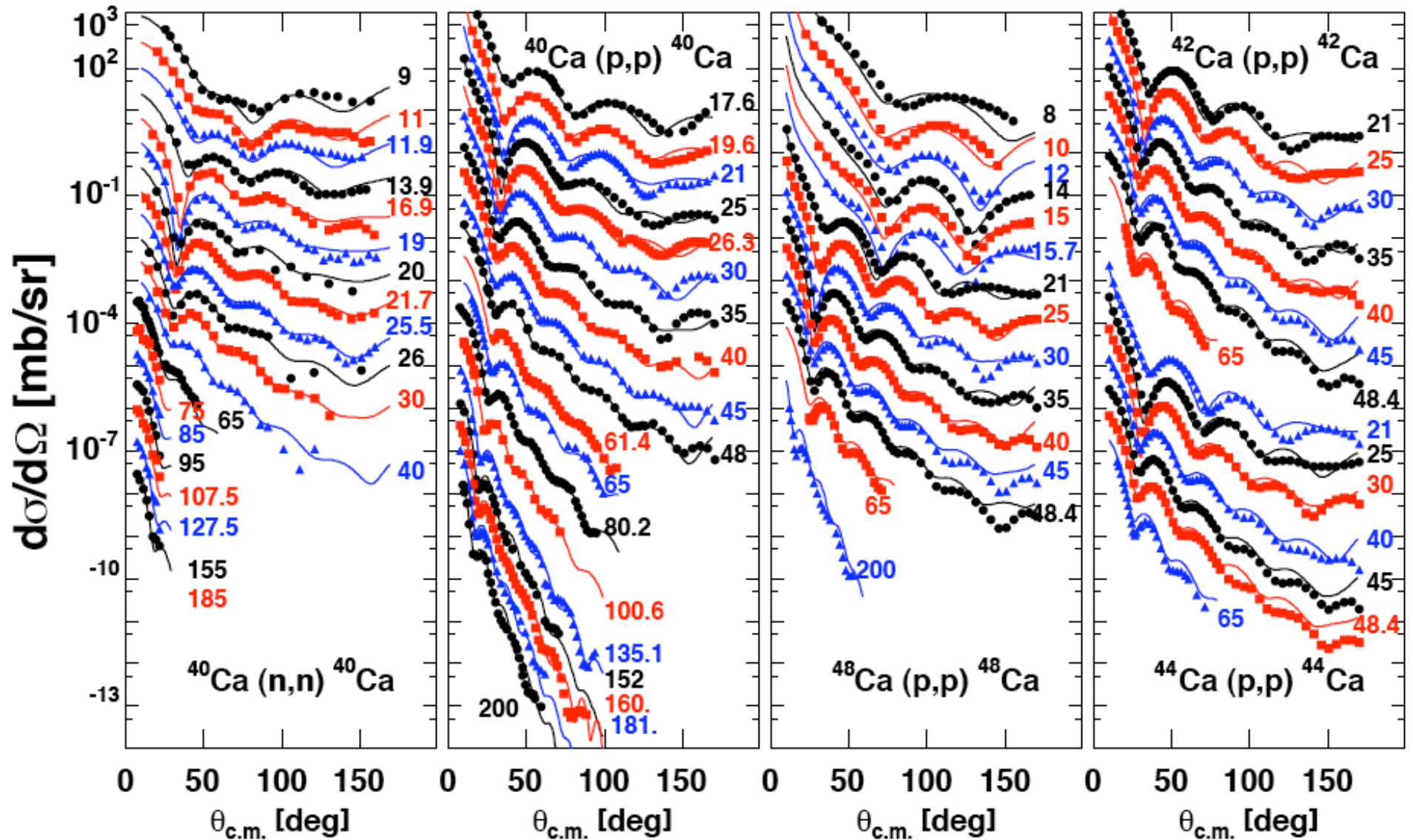
Surface potential strengthens with increasing asymmetry for protons



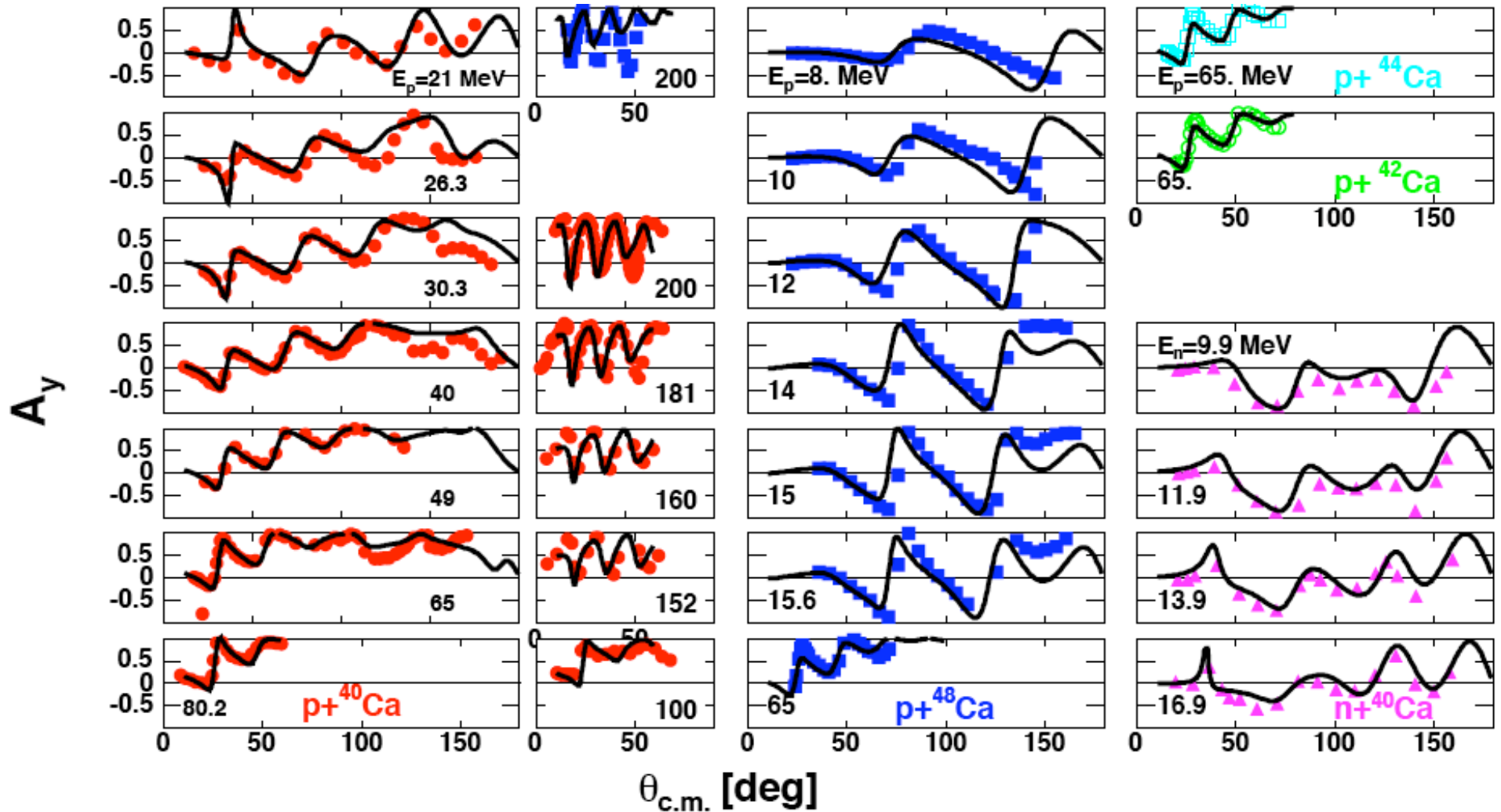
Volume integrals



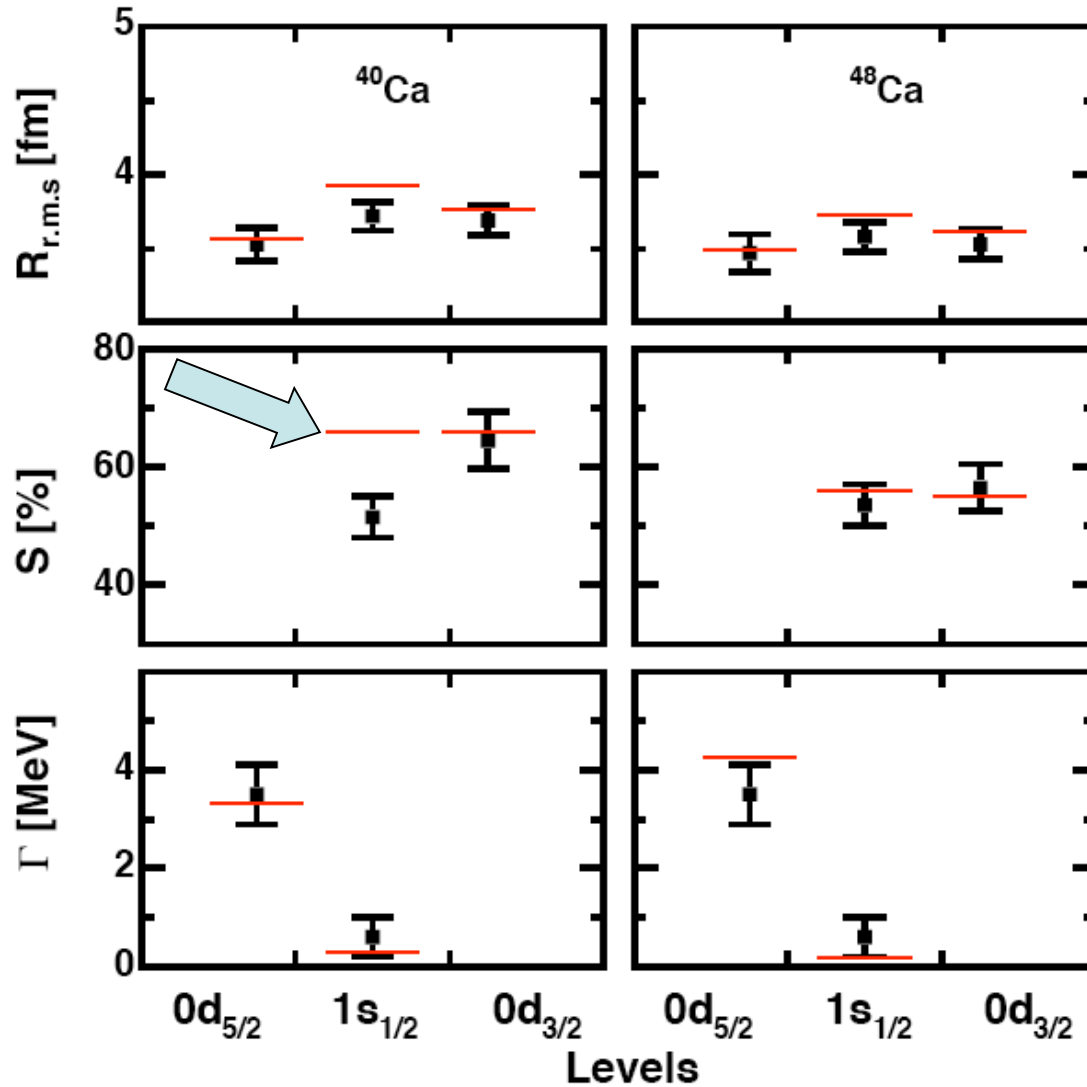
Fit and predictions of n & p elastic scattering cross sections



Present fit and predictions of polarization data



Present fit to (e,e'p) data

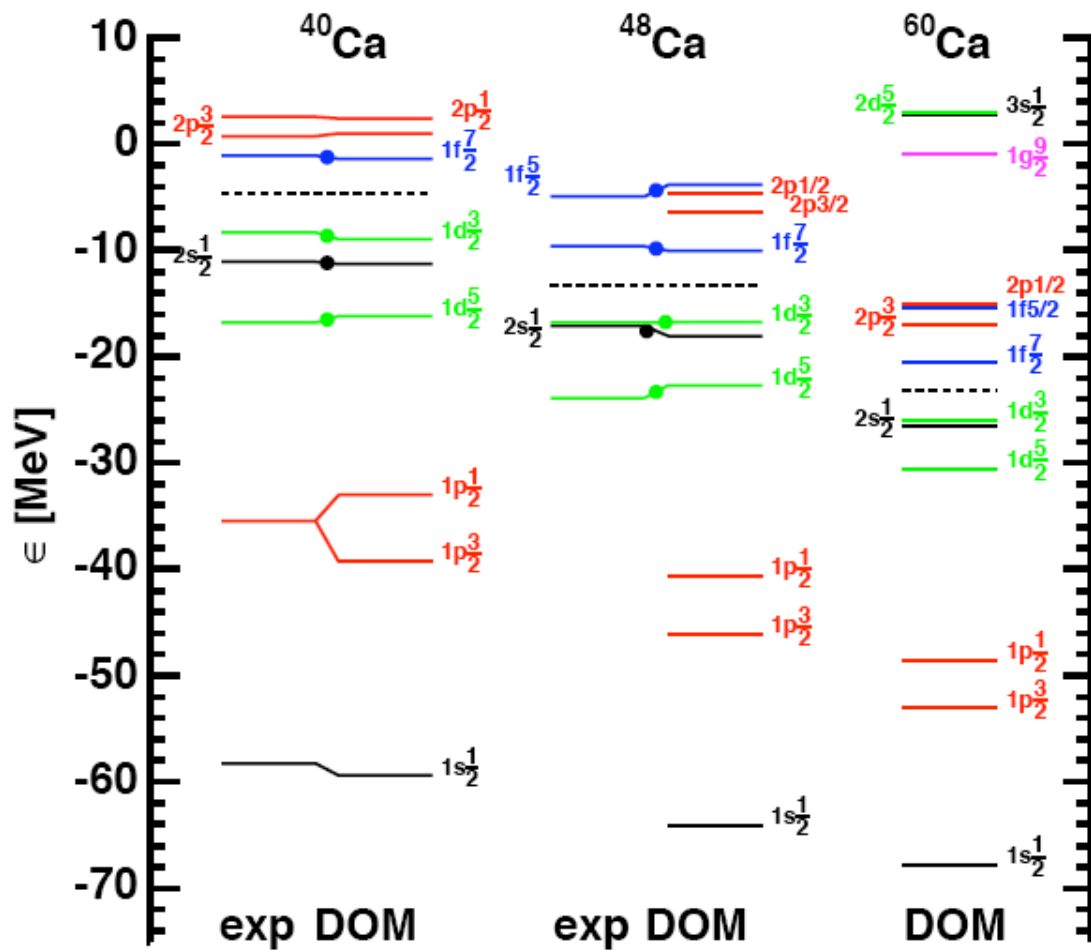


radii of
bound state
wave functions

spectroscopic
factors

widths of strength
distribution

Proton single-particle structure and asymmetry



Pairing of protons due to pn correlations?!

Increased correlations with increasing asymmetry!

Extrapolation in δ

Naive: $p/n \Rightarrow \pm (N-Z)/A$

Cannot be extrapolated for n

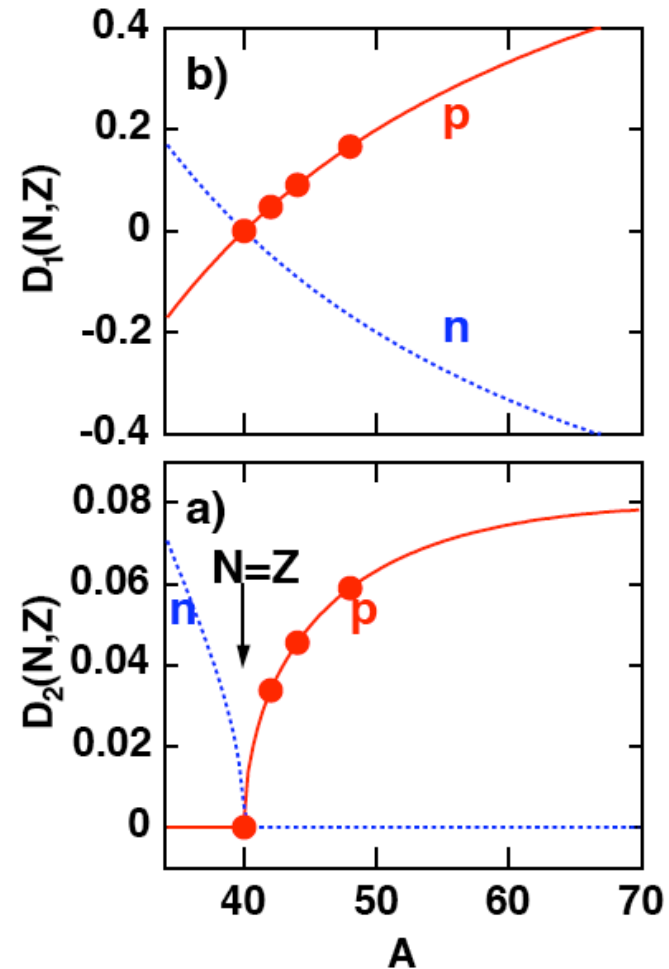
Less naive:

$$U = V_0 + \frac{\vec{t} \cdot \vec{T}}{A} V_1$$

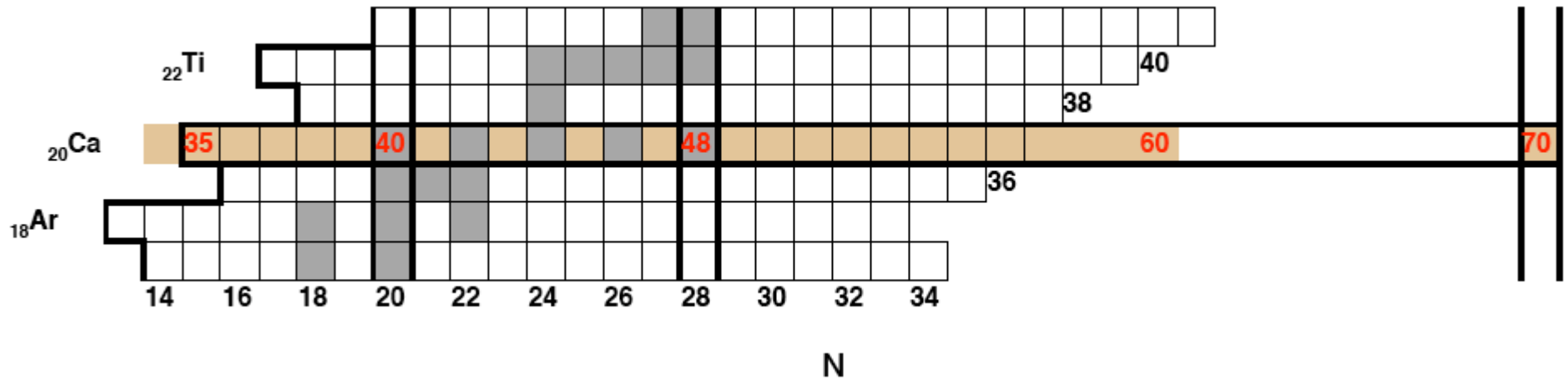
$p/n \Rightarrow \Theta(\pm(N-Z))\sqrt{(|N-Z|)/A}$

Emphasizes coupling to GT resonance

Need $n+^{48}\text{Ca}$ elastic scattering data!!!



Driplines



Proton dripline wrong by 1

Neutron dripline more complicated:

^{60}Ca and ^{70}Ca particle bound
Intermediate isotopes unbound
Reef?

Improvements in progress

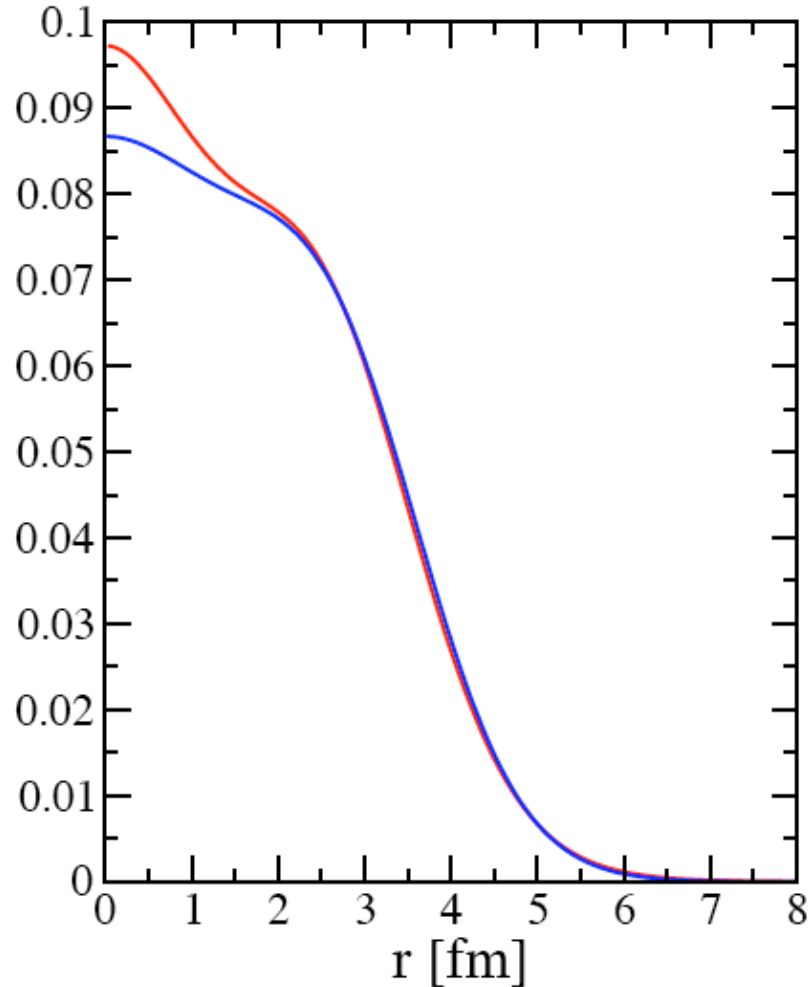
Replace treatment of nonlocality in terms of local equivalent but energy-dependent potential by explicitly nonlocal potential

⇒ Necessary for exact solution of Dyson equation

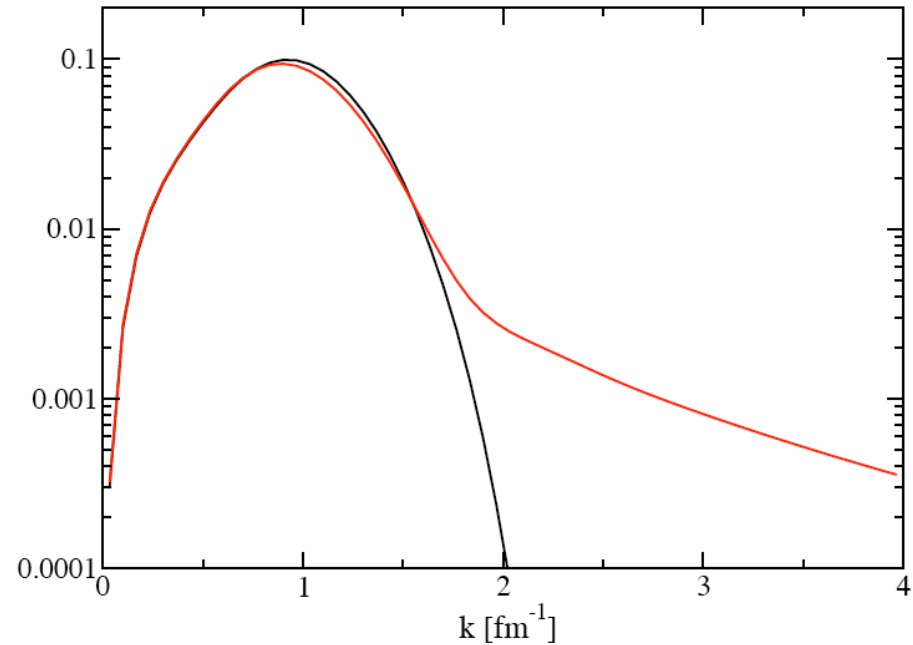
- Yields complete spectral density as a function of energy OK
- Yields one-body density OK
- Yields natural orbits OK
- Yields charge density OK
- Yields neutron density OK
- Data for charge density can be included in fit
- Data for $(e,e'p)$ cross sections near E_F can be included in fit
- High-momentum components can be included (Jlab data)
- E/A can be calculated/ used as constraint ⇒ TNI
- NN Tensor force can be included explicitly
- Generate functionals for QP-DFT

Charge density & High-momentum components

^{40}Ca



$k^2 n(k)$



Only 2% high-momentum strength
 \Rightarrow Modify self-energy to include more
high-momentum strength

Consistent with theoretical experience
and Jlab data!

Summary

- Proton sp properties in stable closed-shell nuclei understood (mostly)

Study of $N \neq Z$ nuclei based on DOM framework and experimental data

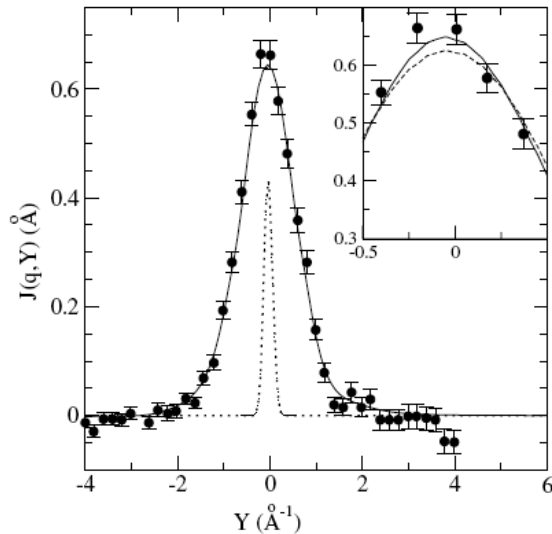
- Description of huge amounts of data
- Sensible extrapolations to systems with large asymmetry
- More data necessary to improve/pin down extrapolation
- More theory

Predictions

- $N \neq Z$ p more correlated while n similar (for $N > Z$) and vice versa
- Proton closed-shells with $N \gg Z \Rightarrow$ may favor pp pairing
- Neutron dripline may be more complicated (reef)

Deep-inelastic neutron scattering off quantum liquids

Liquid ^3He



Response at 19.4 \AA^{-1}

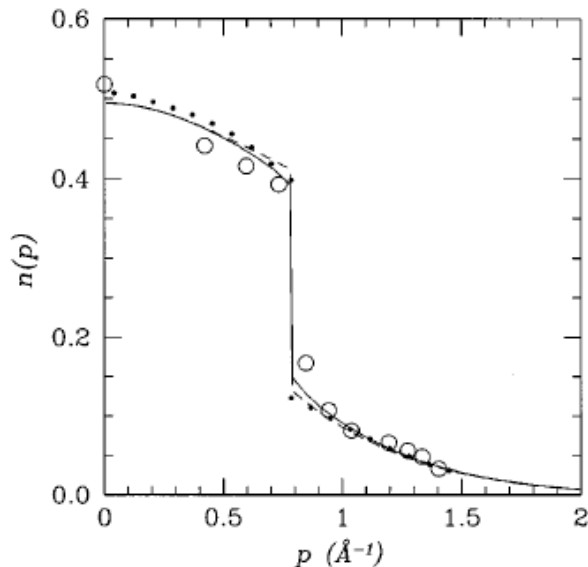
Probe: neutrons

R.T. Azuah et al., J. Low Temp. Phys. **101**, 951 (1995)

Theory: Monte Carlo $n(k)$ & FSE (ρ_2) beyond IA

F. Mazzanti et al., Phys. Rev. Lett. **92**, 085301 (2004)

$$J(Y) = \frac{1}{2\pi^2 \rho} \int_{|Y|}^{\infty} dk k n(k) \quad \text{IA result}$$



$$Y = \frac{m\omega}{q} - \frac{q}{2} \quad \text{scaling variable}$$

Momentum distribution liquid ^3He

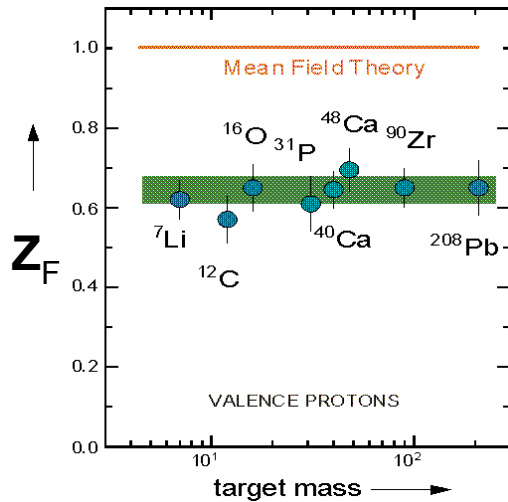
S. Moroni et al., Phys. Rev. B **55**, 1040 (1997)

Comparison of DMC, GFMC, and VMC & HNC

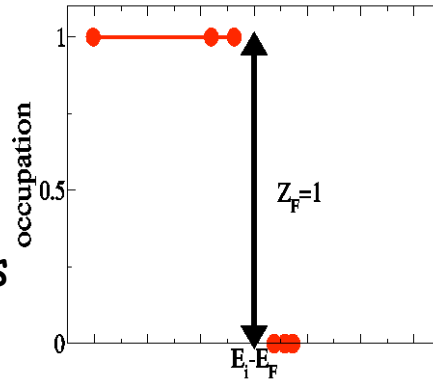
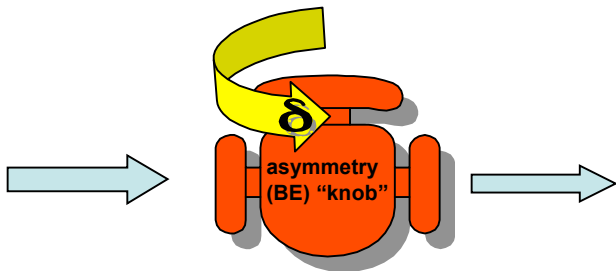
Correlations in ... Atoms

weak correlations

(e,e'p)

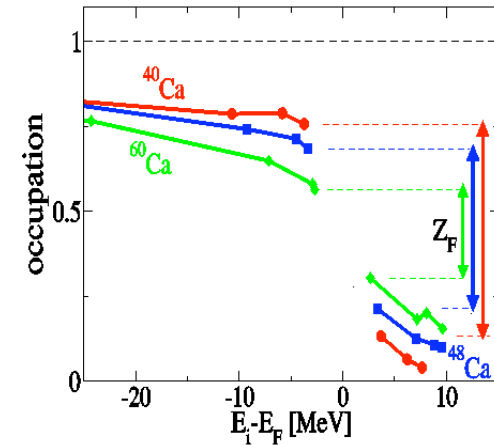
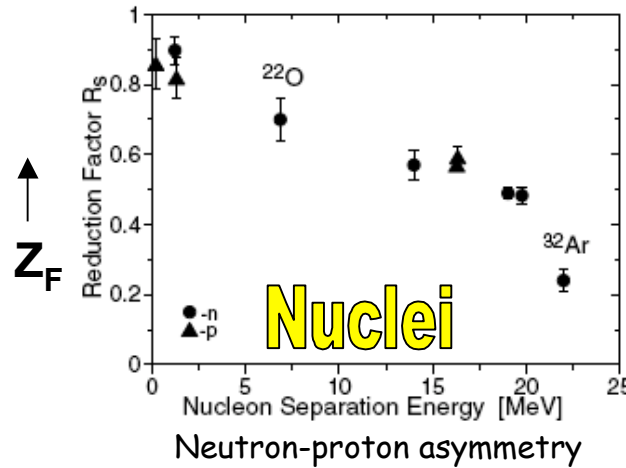


protons in stable
closed-shell nuclei



electrons in Ne
Data from (e,2e)

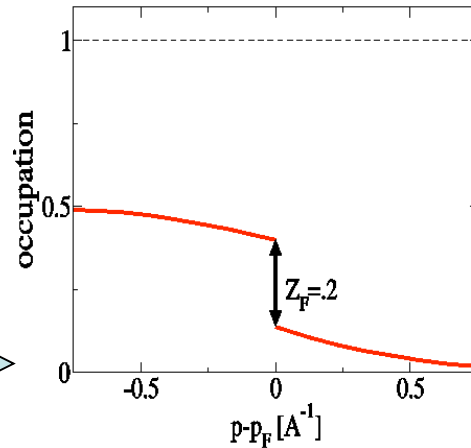
DOM



protons in Ca

Liquid ${}^3\text{He}$

very strong correlations
Data from (n,n')



Nucleon propagator and dripline physics 27