Well-posed continuum equations for granular flow with compressibility and μ(I)-rheology

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ABSTRACT

Continuum modelling of granular flow has been plagued with the issue of ill-posed dynamic equations for a long time. Equations for incompressible, two-dimensional flow based on the Coulomb friction law are ill-posed regardless of the deformation, whereas the rate-dependent μ(I)-rheology is ill-posed when the non-dimensional inertial number I is too high or too low[1]. Here, incorporating ideas from Critical-State Soil Mechanics, we derive conditions for well-posedness of PDEs that combine compressibility with I-dependent rheology. When the I-dependence comes from a specific friction coefficient μ(I), our results show that, with compressibility, the equations are well-posed for all deformation rates provided that μ(I) satisfies certain minimal, physically natural, inequalities[2].

REFERENCES

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[2] Barker, T., Schaeffer, D. G., Shearer, M., and Gray, J.M.N.T., *Well-posed continuum equations for granular flow with compressibility and μ(I)-rheology*, arXiv preprint, arXiv:1610.05135, (2016)