Efficient semi-analytical integration of vortex sheet influence in 3D vortex method

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ABSTRACT

For 3D Lagrangian method based on vortex particles approach, which is suitable for incompressible flow and fluid-structure interaction simulation it is important to calculate vortex sheet influence with high accuracy [1]. The boundary condition is expressed through the integral equation, which is approximated by the system of linear algebraic equations; this approach is similar to Boundary Element Method. It is well-known that there are significant difficulties in numerical evaluation of singular and nonsingular 4-dimensional integrals over triangular panels [2].

In the present paper the original efficient numerical scheme is developed for semi-analytical computation of vortex sheet influence for arbitrary triangular discretization of the body surface. Vortex sheet intensity on the panels assumed to be piecewise-constant. Double integrals over the influence and control triangular panels should be calculated. The integration over the influence panel can be performed exactly, and the resulting formulae are written down in closed form. When the control panels has common edge or common vertex with the influence one, the remaining integrand becomes weakly-singular. In order to compute the corresponding integral with high accuracy it is necessary to exclude the singularity, i.e., to split the integral into regular and singular parts. Regular part is expressed by smooth functions, so it can be easily integrated numerically by using Gaussian quadrature formulae. For singular part exact analytical expressions in closed form is derived.

The numerical scheme is developed for flow simulation around 3D bodies. It is based on tangential components of the velocity boundary condition satisfaction on the body surface instead of widespread condition for normal components. The dimension of the corresponding linear algebraic system is doubled number of triangular panels. The developed approach allows to use arbitrary triangular mesh on body surface and to refine mesh near sharp edges, what is especially important for flow simulation around bodies with complicated geometry.

The developed technique is applied not only for boundary condition approximation, but also for correct simulation of vortex particles motion, which represent vortex wake, especially near the surface.

Several test problems of flow simulation around are considered.

REFERENCES

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