Fusional Method of MPS and FEM through Tetrahedral Mesh Generation

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ABSTRACT

I interpret MPS (moving particle semi-implicit method) as a sort of density method for moving surface. A disadvantage of the density method is that the diffusion of the density calculated by averaging in the element causes difficulty to conserve the mass. In contrast, MPS satisfies the mass conservation. The density method is very suitable for FEM, leading to the concept that "fusional method" would be useful. The tool of special mention in MPS correspond to the weight function to calculate "the particle number density". Since the weight function is a Kernel function, the fusional method uses a novel Kernel function as the weight function of MPS. The masses (particles) occupy the position on vertex node of the tetrahedral elements, and the mesh must be recomposed for the time step advance by using any mesh generation technique, but before mesh recomposition, density ρ is redistributed to satisfy the continuity equation. The method is basically made up of FEM, and I apply Helmholtz-decomposition (abbreviation: *H-d*) using $\nabla\phi$- and $\boldsymbol{u}^T$-elements for displacement in formula $\boldsymbol{u}=\nabla\phi+\boldsymbol{u}^T$ (*T*:*Transverse*).

Kernel function: The particle mass-*i* is allotted as densities by form (1) in inverse ratio of CV (control volume) to individual elements-*j* sharing the node-*i*. (ρ: mass-*i*, *V*: CV for node-*i*, $V_{i,j}$: part-*j* of V)

Kernel function: $$\rho_{j,i} = \frac{\rho}{V}\frac{V_i}{V_{j,i}}, \quad where \quad \frac{1}{V_i} = \sum_j \frac{1}{V_{j,i}} \tag{1}$$

Density function in element-*j*: linear function with parameter values $\rho_{j,i}$ on vertex node-*i*.

Continuity equation: $\dfrac{D\rho}{Dt} + \rho\nabla^2\phi = 0$, where velocity $\boldsymbol{U}$ and $\nabla\phi$ are given, and ρ satisfy the equation.

Navier-Stokes equation: $\mu(\nabla^2 + \dfrac{1}{3}\nabla div)\boldsymbol{U} = \rho\dfrac{D\boldsymbol{U}}{Dt} + \nabla P$, where μ is viscosity of fluid, and *P* is pressure in linear element, and therefor is used to evaluate the incompressibility of the advection term.

Semi-explicit method: Time axially central difference method is used to calculate $\{\boldsymbol{U}, \text{acc.}\}^n$ on time step-*n*, and firstly predict $\boldsymbol{u}^{n+1}$ by Taylor expansion using $\Delta\boldsymbol{x}=\Delta t\boldsymbol{U}$ by iteration method by form (2) to obtain initial values for implicit method, where $\boldsymbol{u}$ is 2nd order element, also to induce surface tension.

Predictor: $$\boldsymbol{u}^{n+1} = \boldsymbol{u}^n + \Delta t\boldsymbol{U}_\text{n}(\frac{\partial\boldsymbol{u}}{\partial\boldsymbol{x}})^n + \frac{\Delta t^2}{2}\boldsymbol{\beta}_\text{n}(\frac{\partial^2\boldsymbol{u}}{\partial\boldsymbol{x}^2})^n \quad \begin{cases} \boldsymbol{U}_\text{n} = \dfrac{\partial\boldsymbol{u}}{\partial t} = \dfrac{\boldsymbol{u}^{n+1}-\boldsymbol{u}^{n-1}}{2\Delta t}, \quad \nabla P + \nabla\boldsymbol{U}_\text{n}\boldsymbol{U}_\text{n} = 0 \\ \boldsymbol{\beta}_\text{n} = \dfrac{\boldsymbol{u}^{n+1}-2\boldsymbol{u}^n+\boldsymbol{u}^{n-1}}{\Delta t^2} + \dfrac{1}{\rho}(\nabla P + \mu\nabla^2\dfrac{\boldsymbol{u}^{n+1}-\boldsymbol{u}^{n-1}}{2\Delta t}) \end{cases} \tag{2}$$

I will explain how to introduce surface tension for variable density in the presentation, also about incompressibility and homogeneousness ($\nabla\nabla^2\phi=0$) of the element.

The proposed method is applicable to the solid, i.e., to solid particles and moving FEM models as well. Thus, the method enables the numerical simulation based on the finite deformation theory.

Equilibrium eq. of solid: $G(\nabla^2 + \dfrac{1}{1-2\nu}\nabla div)\boldsymbol{u} = \rho\dfrac{D\boldsymbol{u}}{Dt}$, where *G* is shearing rigidity, ν is Pisson's ratio and $\boldsymbol{u}$ is displacement vector.

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