A Novel Mixed Meshless Approach for the Strain Gradient Elasticity, Based on the Meshless Local Petro-Galerkin (MLPG) Paradigm

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ABSTRACT

The further development of the mixed meshless Local Petrov-Galerkin (MLPG) paradigm [1] for solving 4^{th}-order differential equations is presented, with application on the strain gradient elasticity. An efficient numerical implementation of gradient elasticity theories in the Finite Element Method is still a non-trivial task, due to the need of solving 4^{th}-order differential equations, and often results in complicated primal C1 algorithms or in mixed formulations based on Lagrangian multipliers [3]. The development of explicit strain gradient C1 models via primal Galerkin meshless methods is straightforward due to a high continuity order of meshless functions [4], but such approaches are plagued by high numerical costs and reduced accuracy due to high order of derivatives [2, 5]. In mixed meshless approaches, complications may arise due to the imposition of boundary conditions [2], large global systems of equations [6] or complicated assumed gradient fields [7].

Herein, the existing mixed MLPG meshless stratagem [1] is modified in order to better address these problems. Besides displacements, higher-order displacement gradients up to the 3^{rd} order are approximated. Governing equations are based on the local weak forms of the principles of conservation of linear and angular momentum and compatibility conditions between approximated variables. The proposed mixed methods do not employs Lagrangian multipliers and all variables are approximated by same meshless functions, such as the Moving Least Squares approximation. By choosing appropriate test functions, various methods may be derived, including a meshless counterpart of the Mixed Finite Volume Method [8]. The use of the mixed approach lowers the continuity requirement on trial functions and enables the use of low-order polynomial bases with small supports, improving numerical efficiency in terms of computational costs and stability. Further improvement is obtained by imposing compatibility conditions by collocations and/or strain smoothing. The validity of above statements is corroborated by preliminary results of numerical benchmark tests.

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