Numerical Scheme for the Finite Pointset Method to Solve Transport Equations on Fixed Pointclouds in 3D

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ABSTRACT

The Finite Pointset Method (FPM) [1] is a meshfree particle method to solve fluid dynamic equations. The spatial derivatives are approximated by the weighted least squares method. If all particles are distributed on a Cartesian grid the classical central differences will be obtained. Therefore FPM can be described as a generalized finite difference method. Although FPM is a Lagrangian method, where the particles move with fluid velocity and carry the information of all physical entities which are needed, FPM can also be used as an Eulerian method, where the particles are fixed. Using the Lagrangian method the CFL condition has always to be fulfilled because a particle must not overtake another particle. But there are many applications with significant local refinement. Hence, the discrepancy between the numerical time step size and the time to be covered by the simulation becomes critical. Moreover, if the steady state of a solution is in demand it will be almost impossible to be solved by the Lagrangian method. Therefore in such applications the Eulerian method is required because it enables bigger time step sizes and a better way to access stationary solutions. In combination with implicit time integration methods even arbitrary large time step sizes are possible. The biggest problem that the Eulerian method offers is the numerical treatment of transport. While in Lagrangian methods transport proceeds by the movement of particles, in the Eulerian formulation a nonlinear transport term must be approximated and numerically solved. Therefore, in this paper we focus on the development of a numerical scheme for FPM to solve transport equations on fixed pointclouds.

Due to the central difference character of FPM it is impossible to approximate the transport term directly with the classical FPM operator because of stability reasons. Hence for a stable discretization it is necessary to calculate a one-sided difference in an upwind-manner. To obtain that a numerical flux function of finite volume schemes is used. The idea is to construct a locally rotated coordinate system which yields a main direction along which the numerical flux function can be used [2]. Along the remaining directions, the use of central differences is still sufficient. This approach yields an upwind scheme which is extremely diffusive. To improve the accuracy, reconstruction techniques are applied [2].

Since the main issue is the spatial discretization of nonlinear transport terms, all tests are computed with explicit time integration methods conveniently. The approach is evaluated by reference to the simple transport of a cube which shows high accuracy of the new scheme. As an industrial application salt mining is presented whereby the interrogation relates to the development of the salt cavern over many years. This application clarifies the importance of the Eulerian method because it is impossible to solve this problem with the Lagrangian method. To compare both methods the industrial application is simplified.

REFERENCES

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