Mixed finite elements for accurate prediction of strain localization failure in geotechnical related problems

Lorenzo Benedetti*, Miguel Cervera* and Michele Chiumenti*

* Centre Internacional de Mètodes Numèrics en Enginyeria (CIMNE)
Universitat Politècnica de Catalunya (UPC)
Campus Norte UPC, 08034 Barcelona, Spain
e-mail: congreso@cimne.upc.edu, web page: http://www.cimne.upc.edu/

ABSTRACT

Determination of the initial detaching volume is one of the major challenges in landslides, debris flow prediction and snow avalanches. In the catastrophic failure of slopes, the creation of a slip line in the soil triggers the release of a mass that may develop into a landslide or a rockslide. On the other hand, quasi-brittle failure triggers the release of snow avalanches. Induced strain softening is the main cause of failure in both cases.

In the recent decades, the finite element method has established as the preferred choice for stability analyses, thanks to the possibility of describing complex geometries, different materials and diversified boundary conditions. However, the problem of strain localization poses numerous challenges to be overcome. Within the framework of standard irreducible finite elements, the smeared crack approach [1] allows treating discontinuity as a band of finite width, where the displacements are continuous and the strains are discontinuous, but bounded. Nevertheless, this hypothesis is well-known to present serious numerical drawbacks. In fact, spurious mesh dependence can occur and the fracture line direction is consequently biased. Moreover, when isochoric behaviour is enforced (as in the case of undrained soil), locking of the stresses provokes pressure oscillations, with the resulting pollution of numerical calculations. Both problems can be shown not to be related to the mathematical statement of the continuous problem but, instead, to its discrete (FEM) counterpart.

Recently, the authors proved that strain localization numerical issues can be easily alleviated using a Mixed Finite Element formulation, in terms of strain and displacements [2]. In particular Badia and Codina [3] and, then, by Cervera et al. [4], reported that the order of convergence of strains (and stresses) in mixed formulations is one order higher than the displacement-based method, even in case of localized discontinuities. The strain-displacement mixed finite element formulation provides enhanced stress accuracy for a given mesh and allows the determination of localization bands without the introduction of auxiliary tracking techniques. Both compressible and incompressible plasticity are tackled in [5], where, furthermore, it was shown that the energy dissipation is exactly matched in the case J2 plasticity, confirming the consistency of the method.

From these premises, various problems of geotechnical interest are tackled to demonstrate the capabilities of the formulation. On the one hand, the standard irreducible formulation is compared with the introduced mixed formulation in benchmark problems aimed to show the achieved improvements. On the other hand, the method is used to provide examples of case studies, involving stability of slopes, shallow foundations and release of snow avalanches.

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