

# **Coulomb friction from Newtonian restitution - with help from J.-J. Moreau and D.G.B. Edelen**

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## **ABSTRACT**

According to recent numerical simulations [1, 5], assemblies of frictionless spheres and polygons exhibit a flowing “yield” stress but no dilatancy. These findings, based on Newtonian dynamics, are at odds with certain quasi-static simulations [3] that tend to confirm the well-known hypothesis of O. Reynolds. According to the latter, dilatancy represents a “powerless” reactive stress arising from the coupling of volume change to shape change (deviatoric shearing). The failure of the quasi-static description can be attributed to the collapse of local, unstable dilated states, with dissipation of volumetric strain energy.

In the simulations of [1, 5], the dissipation of volumetric energy is represented by collisional energy loss, as modeled by a Newtonian coefficient of restitution less than unity. However, other models of rate-independent plasticity in molecular solids [6] and dry foams [7] suggest that any number of models of rate-dependent dissipation would produce a similar result. The general idea is that all these phenomena involve transitions over barriers in a potential-energy landscape, with dissipation of energy on vanishingly small time scales. In the case of cohesionless granular media, the potential energy can be attributed to “p-V” work against an external confining pressure, in accordance with the ideas of Reynolds.

This talk presents an attempt to fit all this into a coherent theoretical framework based on the notion of the convex dissipation potentials postulated by J.-J. Moreau [4] and confirmed by D.G.B. Edelen [2].

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