Error Estimates of a Particle Based Method Based on a Weak Formulation

TAGAMI, Daisuke*

* Institute of Mathematic for Industry (IMI), Kyushu University
744 Motooka, Nishi-ku, Fukuoka, 8190395, Japan
e-mail: tagami@imi.kyushu-u.ac.jp, web page: http://www2.imi.kyushu-u.ac.jp/~tagami/

ABSTRACT

As far as we know, there are few error analysis of particle based methods like Smoothed Particle Hydrodynamics (SPH) method and Moving Particle Semi-implicit (MPS) method. We can find few example of related results; Raviart has established error analysis of vortex methods for the vortex equation [1], Ishijima-Kimura have established truncation error analysis of finite difference operators appearing in MPS method [2], Ben Moussa have established convergence analysis of SPH method for multidimensional conservation laws [3], and some papers cited in there references.

Recently, we have obtained error estimates of a generalized particle based method for Poisson equations and heat equations [4]. Our generalized method can describe the interpolants and the approximate differential operators in case of not only SPH or MPS but also other methods, where it is possible to choose various weight functions not considered in the conventional ones. In [4], *regular* particle distributions with respect to the covering radius and the influence radius and *conectivity* of the particles play important roles in our estimates. By extending the results in [4], we obtain truncation error estimates with appropriate norms appearing in a particle base method based on a weak formulation. Moreover, by introducing analogies like domain decomposition methods, we define a particle based method based on weak formulations in an extended domain, and establish error estimates of it.

Finally, some numerical results are shown. The numerical convergence rates of errors agree well with the mathematical ones.

REFERENCES

[1] Raviart, P.-A., “An analysis of particle methods”, *Numerical Methods in Fluid Dynamics*, *Lecture Notes in Mathematics*, **1127**, 243-324, (1985).

[2] Ishijima, K. and Kimura, M., “Truncation error analysis of finite difference formulae in meshfree particle methods”, *Transactions of the Japan Society for Industrial and Applied Mathematics*, **20**, 165-182, (2010).

[3] Ben Moussa, B., “On the convergence of SPH method for scalar consevation laws with boundary conditions”, *Methods and Applications of Analysis*, **13**, 29-62, (2006).

[4] Imoto, Y. and Tagami, D., “Error estimates of a particle-based methods for Poisson equations”, Proceedings of JSIAM annual meeting, (2014), *in Japanese*.