

Jean-Jacques Moreau's contributions to Granular Mechanics

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ABSTRACT

Jean-Jacques Moreau (1923-2014) is well known to the community of mathematicians for having initiated Convex Analysis together and independently with R. Tyrrell Rockafellar. He has written many papers containing original and important theorems and concepts: the decomposition of a vector on two polar cones, properties of measures on the real line, the sweeping process.... He usually introduced himself as a mechanic claiming that he was doing mathematics just enough for mechanical purposes. He has published many papers in fluid mechanics and solid mechanics. He became interested in granular materials in the eighties, mainly from the numerical point of view, while being much concerned with the mathematical aspects and much aware of engineering problems. The sweeping process associated with the catching up numerical algorithm is the opening key to frictional contact problems. This process motivated at the beginning by plasticity is a prototype of an evolution conditioned by inequality constraints, and thus it has straightforward applications to unilateral dynamics, namely sweeping a particle by a moving convex set. J. J. Moreau exhibited how non regular inelastic shock laws and Coulomb friction law could be expressed in terms of Convex Analysis, in which one finds the classical variational inequalities formulations. Since these concepts are relevant, they should suggest algorithms to solve frictional contact problems. To demonstrate this idea, he numerically solved successfully the simple problem of a particle meeting an obstacle with a shock law and Coulomb friction (1984). The algorithm (Contact Dynamics method) was soon extended to collections of rigid bodies and to deformable bodies by other contributors (Non Smooth Contact Dynamics). The main features of this CD algorithms is that they are fully implicit algorithms allowing one to use large time steps. It is thus a new approach different from the classical Molecular Dynamics explicit methods (see P. Cundall & al.) feasible with smooth (or regularized) frictional contact laws and small time steps. J. J. Moreau had his own style of programming and presenting results with graphical tools also of his own. He investigated controversial questions as avalanches, segregation of particles under vibrations, vault effects in a stack of particles, locking and indeterminacy..., discussed in papers and also presented as pedagogical films. He also contributed significantly to the mechanics of monuments made of blocks modeled just as particles with a particular shape. He has been much concerned by the question of stress in granular materials. He numerically demonstrated that the concept of stress in a granular material (like the Cauchy stress in continuous media) is relevant. The usual definition of the stress in a granular material produces a slightly asymmetric tensor. Around 2008 he gave the exact definition of what should be a stress tensor in a granular material, and even more so in any collections of bodies. Finally, by defining a divergence operator, he showed that the equilibrium equation in such collections may be written $\text{div}\sigma = f$, like in any continuous medium.

Many contributions and references can be found in the following books :

- 1) Micromechanics of granular materials edited by Bernard Cambou, Michel Jean and Farhang Radjai, ISTE 2009.
- 2) Matériaux granulaires : modélisation et simulation numérique sous la direction de Bernard Cambou, Michel Jean, Farhang Radjai, 2012, Lavoisier.