Solution of the stationary Stokes and Navier-Stokes equations using the Modified Finite Particle Method in the framework of a Least Square Residual Method

D. Asprone[1], F. Auricchio[2,3], A. Montanino[2,3,]* and A. Reali[2]

[1]Dipartimento di Strutture per l'Ingegneria e l'Architettura (DIST) - Università di Napoli Federico II
Via Claudio, 21 – 80125 Napoli (ITALY)

[2]Dipartimento di Ingegneria Civile ed Architettura (DICAr) - Università di Pavia
Via A. Ferrata, 3 – 27100 Pavia (ITALY)

[3]Centro di Simulazione Numerica Avanzata (CeSNA) - Istituto Universitario di Studi Superiori
Piazza della Vittoria, 15 – 27100 Pavia (ITALY)

ABSTRACT

The numerical solution of equations that model elastic incompressible fluids represents a challenge for many approximation methods, due to the existence of a numerical limitation, first investigated in [1], which results in the so called inf-sup condition. When the inf-sup condition is not respected, the pressure exhibits non-physical oscillations, known in the literature as checkerboard instability.

In the present work we attack incompressible elasticity problems using the Modified Finite Particle Method (MFPM) [2-3] for the spatial discretization in the framework of the Least Square Residual Method (LSRM) [4]. This approach permits to discretize in the same way velocity and pressure fields, without violating the inf-sup condition.

The MFPM is a numerical procedure for function and derivative approximations, based on the Taylor series expansion. It is a **local** method, since approximations are based on the nodes in the neighborhood of each collocation point; it is a **meshless** method since there is no *a priori* connectivity between nodes; finally, it is a **collocation** method, since partial differential equations are approximated in their strong form, avoiding problems related to numerical integrations.

The LSRM is a technique for the solution of linear systems where the number of equations is greater than the number of unknowns. When the system is the discrete form of a partial differential problem, it means that the number of collocation points is greater than the number of nodal unknowns. The solution of the system is then found by minimizing the squared error $E = (\boldsymbol{K}\hat{\boldsymbol{u}} - \boldsymbol{f})^{\boldsymbol{T}}(\boldsymbol{K}\hat{\boldsymbol{u}} - \boldsymbol{f})$

Some weights are introduced in the formulation in order to restore the same error magnitude from field equations and from boundary conditions or incompressibility constrain. The final system to be solved is thus $\boldsymbol{K}^T\boldsymbol{W}\boldsymbol{K}\,\hat{\boldsymbol{u}} = \boldsymbol{K}^T\boldsymbol{W}\boldsymbol{f}$ that is a symmetric system of equations (not common in collocation methods). The results that we obtain combining MFPM and LSRM confirm the effectiveness of the method, in the linear and in the non-linear case. Where an analytical solution of problems is available, we also show that second-order accuracy of the method is achieved.

REFERENCES

[1] Brezzi, F. and M. Fortin (1991). *Mixed and hybrid finite element methods*. Springer-Verlag New York, Inc.

[2] Asprone, D., F. Auricchio, and A. Reali (2014). Modified Finite Particle Method: applications to elasticity and plasticity problems. *International Journal of Computational Methods 11*(01).

[3] Asprone, D., F. Auricchio, A. Montanino, and A. Reali (2014). A Modified Finite Particle Method: Multi-dimensional elasto-statics and dynamics. *International Journal for Numerical Methods in Engineering*.

[4] Chi, S.-W., J.-S. Chen, and H.-Y. Hu (2014). A weighted collocation on the strong form with mixed radial basis approximations for incompressible linear elasticity. *ComputationalMechanics 53*(2),309–324

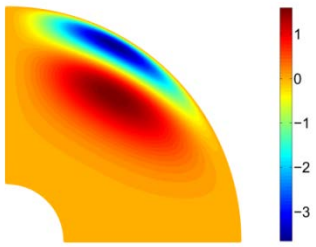

Fig.1 Incompressible quarter of annulus under body loads: horizontal displacements obtained with MFPM

Fig.2 The lid-driven cavity problem: streamlines obtained with MFPM for Re=400