A continuous outlook of discrete mechanics

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The quasistatic flow of rigid frictional particles can be seen as a succession of unstable transitions between equilibrium states at both particles and system scales. Each stage of the flow is characterized by a set of internal parameters encoded in the geometry of the packing (packing fraction, coordination number, fabric tensor, \ldots). Given the stress conditions, the material yields and reorganizes itself towards a stable state under a set of admissible values of these parameters. The rheology of granular materials consists thus of relating the equilibrium properties (i.e. the stress states a local arrangement of particles can support without yielding) to the values of internal parameters. In order to characterize this rheology, it is proposed to follow a discrete element simulation in parallel with a continuum description slaved to the former through spatial averaging operators for different resolutions both in time and space. By using this method, we obtain fields for both relevant internal variables and state variables (momentum, stress, strain) which are compatible with continuum mechanics. This approach allows us to quantify the effects of spatio-temporal fluctuations of state variables to the rheological behavior and therefore to address the relevance of the statistical nature of the granular assembly at the scale of the whole system.





Figure 1: (Top): Instantaneous particle velocity vectors in a homogeneous quasistatic shear simulation (Bottom): corresponding continuum velocity field (norm of the velocity) based on averaging operators at the lowest spatial and temporal scales.