Error estimates of a particle-based method for partial differential equations

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ABSTRACT

Error estimates of a particle-based method for partial differential equations, the Poisson equations and the heat equations, are derived. Our method is a class of particle-based ones, which describes not only Smoothed Particle Hydrodynamics (SPH) or Moving Particle Simulation (MPS) but also other methods to choose various weight functions not considered in the conventional ones.

There are few literatures for error estimates of the particle-based method. For example, Mas-Gallic–Raviart and Moussa–Via established error estimates of SPH for conservation lows [1, 2]. However, SPH used in [1, 2] are little different from SPH in practical computational fluid dynamics. For example, the particle volume in [1, 2] is defined by the Jacobian derived from flow fields, which is not appropriate for practical computations.

First, we introduce a class of particle-based method for the Poisson equations proposed in [3]. By introducing regularity and connectivity conditions of particle distributions and influence radius, we obtain the unique solvability and the maximum principle of the discrete problem. By these facts, we establish error estimates, which are optimal in the sense that its have the same convergence orders as the orders of interpolants with respect to the influence radius. Next, we analyze a class of particle-based method for the heat equations. Under the regularity condition of the particle distributions and the influence radius, we establish optimal error estimates with restrictions between influence radius and time increments. Finally, some numerical results are shown. The numerical convergence rates of errors coincide well with the mathematical ones.

REFERENCES

[1] S. Mas-Gallic and P. A. Raviart, "A Particle Method for First-order Symmetric Systems", Numerische Mathematik, 51.3, 323-352 (1987).
[2] B. B. Moussa and J. P. Vila, "Convergence of SPH Method for Scalar Nonlinear Conservation Laws", SIAM Journal on Numerical Analysis, 37.3, 863-887 (2000).
[3] Y. Imoto and D. Tagami, "Error estimates of a particle-based methods for Poisson equations (in Japanese)", JSIAM annual meeting abstracts (2014).