

A depth-averaged $\mu(I)$ -rheology for shallow granular free-surface flows

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ABSTRACT

The $\mu(I)$ -rheology is a nonlinear viscous law, with a strain-rate invariant and pressure-dependent viscosity, that has proved to be effective at modelling dry granular flows in the intermediate range of the inertial number, I . This paper shows how to incorporate the rheology into depth-averaged granular avalanche models. To leading order, the rheology generates an effective basal friction, which is equivalent to a rough bed friction law. A depth-averaged viscous-like term can be derived by integrating the in-plane deviatoric stress through the avalanche depth, using pressure and velocity profiles from a steady-uniform solution to the full $\mu(I)$ -rheology. The resulting viscosity is proportional to the thickness to the three halves power, with a coefficient of proportionality that is angle dependent. When substituted into the depth-averaged momentum balance this term generates second-order derivatives of the depth-averaged velocity, which are multiplied by a small parameter. Its inclusion therefore represents a singular perturbation to the equations. It is shown that a granular front propagating down a rough inclined plane is completely unaffected by the rheology, but, discontinuities, which naturally develop in inviscid roll-wave solutions, are smoothed out. By comparison with existing experimental data, it is shown that the depth-averaged $\mu(I)$ -rheology accurately predicts the growth rate of spatial instabilities to steady-uniform flow, as well as the dependence of the cut-off frequency on the Froude number and inclination angle. This provides strong evidence that, in the steady-uniform flow regime, the predicted decrease in the viscosity with increasing slope is correct. Outside the range of angles where steady-uniform flows develop, the viscosity becomes negative, which implies that the equations are ill-posed. This is a signature of the ill-posedness of the full $\mu(I)$ -rheology at both high and low inertial numbers. The depth-averaged $\mu(I)$ -rheology therefore cannot be used outside the valid range of angles without additional regularization.

REFERENCES

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