Development of higher order particle discretization scheme for analysis of failure phenomena

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ABSTRACT

We consider the higher order extension of Particle Discretization Scheme (PDS)[1] and its implementation in the FEM framework (PDS-FEM)[1]. PDS-FEM provides simple and numerically efficient treatment for modelling cracks and has been applied to simulate various linear, non-linear and dynamic crack propagation phenomena in 3D. A unique property of PDS is that is uses conjugate tessellations (e.g. Voronoi $\{\Phi^{\alpha}\}$ and Delaunay $\{\Psi^{\beta}\}$ tessellations pair) to approximate functions and their derivatives. In the original formulation, characteristic functions of Voronoi and Delaunay tessellations (e.g. $\{\phi^{\alpha}(x)\}$ and $\{\psi^{\beta}(x)\}$) are used for approximating functions and derivatives, respectively. It is this use of non-overlapping characteristic functions which facilitate the simple and numerically efficient failure treatment.

In the higher order extension of PDS, a function and its derivatives are approximated as a summation of local polynomial expansions around mother points of the elements of the tessellations, instead of characteristic functions. As an example, a function $f(x)$ and its derivative $\mathrm{g}(x)$ are approximated as $f^{d}(\boldsymbol{x}) \cong \sum_{\alpha,n} f^{\alpha_n} P^{\alpha_n}$ and $g^{d}(\boldsymbol{x}) \cong \sum_{\beta,n} g^{\beta_n} P^{\beta_n}$, where $\{P^{\alpha_n}\} = \{1, x - x^{\alpha}, \dots\}\varphi^{\alpha}(x)$ and $\{P^{\beta_n}\} = \{1, x - x^{\beta}, \dots\}\psi^{\beta}(x)$ are the local polynomial bases with the respective tessellation element as their support. $f^{d}(x)$ consists of numerous discontinuities since the supports of local polynomial bases, P^{α_n}'s, are the non-overlapping tessellation elements Φ^{α}'s. Also, the use of polynomial bases on non-overlapping supports gives rise to the particle nature. It is to smoothly connect those discontinuous local polynomial expansions and define a bounded approximation, the derivative $\mathrm{g}(x)$ is approximated on the conjugate tessellation $\{\Psi^{\beta}\}$.

The proposed higher order extension of PDS is implemented in FEM framework, improving the accuracy of PDS-FEM. Validation and verification of proposed higher order PDS-FEM has been done with different benchmark problems. With the classical plate with a hole problem, it is demonstrated that second order accuracy in the displacement field can be attained with the polynomial bases up to the first order (i.e. $\{1, x - x^{\alpha}\}$ and $\{1, x - x^{\beta}\}$). Further, J-integral about a mode-I crack tip files is estimated to demonstrate the improvement in accuracy of crack tip stress fields..

REFERENCES

[1] Muneo Hori, Kenji Oguni and Hide Sakaguchi, "Proposal of FEM implemented with particle discretization scheme for analysis of failure phenomena", *Journal of Mechanics and Physics of Solids*, **53**, 681--703 (2005).