Solutions of Young–Laplace equation and stability analysis

Olivier Millet, Boleslaw Mielniczuk, Gérard Gagneux, Rafik Belarbi

LaSIE, Université de La Rochelle (France)
Emails : olivier.millet@univ-lr.fr, boleslaw.mielniczuk@univ-lr.fr, gerard.maryse.gagneux@gmail.com, rafik.belarbi@univ-lr.fr

ABSTRACT

In this work, we propose an original resolution of Young–Laplace equation for capillary doublets from an inverse problem. The missing information on the pressure deficiency (which is often an unknown of the problem) will be restored without experimental device of suction control. Only the use of a digital camera with macrozoom allows to measure the suction according to the observed value of the gorge radius, of the filling and wetting angles.

We establish a simple explicit criterion based on the observation of the contact point, the wetting angle and the gorge radius, to classify in an exhaustive way the nature of the surface of revolution: portion of nodoid, of unduloid, both with concave or convex meridian, of catenoid, of cylinder or of circular profiles (toroid). In every case, we propose an exact parametric representation of the meridian based on the observed geometry of the boundary conditions. This method is practical and well efficient for calculating the binding forces, areas and volumes. It avoids to have recourse to the simple toroidal approximation or to spline functions that do not respect (except in an exceptional theoretical case) the Young–Laplace equation. Moreover, we prove that the inter-particle force may be evaluated on any section of the capillary bridge and constitutes a specific invariant of Young-Laplace equation.

The pertinence of the addressed approach will be put in a prominent position on several experimental results obtained on various geometries of capillary bridges. Moreover, the stability of solutions of Young-Laplace equations will be analyzed, based on the second variation criterion of the associated potential (minimization problem under constraints), revisited through Vogel's stability criteria. A theoretical stability criterion and conjectures on breakage will be proposed and discussed.