Regularising Granular Flow Rheology

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ABSTRACT

In light of the successes of the Navier--Stokes equations in the study of fluid flows, similar continuum treatment of granular materials is a long standing ambition. Historically this has been attempted through modification of the dissipation terms in the momentum balance equations, effectively introducing pressure and strain-rate dependence into the viscosity. Originally a popular model for this granular viscosity, the Coulomb rheology, proposed rate-independent plastic behaviour scaled by a constant friction coefficient μ. Unfortunately, the resultant equations are always ill-posed [1]. Mathematically ill-posed problems suffer from unbounded growth of short wavelength perturbations, which necessarily leads to grid dependent numerical results that do not converge as the spatial resolution is enhanced. As such, it is vital to seek well-posed equations to make physically realistic predictions.

The recent $\mu(I)$-rheology [2] is a major step forward as it includes rate dependence through the non-dimensional inertial number I. We show that the resultant equations are well-posed for intermediate values of I, but ill-posed for both high and low inertial numbers [3]. This result is not obvious from casual inspection of the equations, and suggests that additional physics, such as enduring force chains and binary collisions, become important in these limits. The theoretical results are validated numerically using two implicit schemes for non-Newtonian flows.

We also consider the implications of the analysis to extended rheological models such as those found to fit experimental data more accurately in the quasi-static limit [4]. It is found that these theories broaden the validity of the equations further but still break down in the extreme inertial limits. As such, we make use of the stability criterion to motivate a rheology that remains well-posed over the greatest possible range of flow regimes. This regularised model is then tested for problems that were previously hindered by the ill-posedness.

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