Development of a Normal and Tangential Elastic-Plastic Force-Displacement Model For Discrete Element Simulations

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ABSTRACT

We present an accurate model for the normal and tangential force-displacement relationships between elastic-plastic spheres in contact for use in discrete element method (DEM) simulations. For normal forces the model uses a piecewise constant approach with distinct expressions for the elastic loading, elastic-plastic loading and unloading. For tangential forces the model is based on the Langston, Tüzün, and Heyes (LTH) model[1]. It is developed by analysing the force-displacement response during a collision between elastic-perfectly plastic spheres and a rigid surface using detailed finite element method (FEM) simulations.

The normal elastic loading is given by the Hertz model[2] up to a yield displacement as defined by Thornton[3]. By considering the two fundamental aspects of contact: normal contact pressure and the radius of the contact area a novel expression for normal elastic-plastic loading is developed. For normal unloading an adapted version of the Hertz model is used with an increased effective radius of curvature to account for the permanent plastic deformation of the particles. Empirical relationships are found that relate the parameters of the new model to material properties. This allows the model to be used in the DEM for direct simulation of well-characterised elastic-plastic materials without fitting parameters to experimental results. This gives the model an advantage over models in the literature for which fitting to experimental results is required.

The model is validated against the results from FEM simulations. The new model shows a good match to the FEM results and the DEM implementation correctly distinguishes between the loading, unloading and re-loading phases of contact between two spheres. It also performs better than existing models to which it is compared.

The model has been implemented into the DEM code MultiFlow. Large scale DEM simulations have been carried out of the bulk compression of well-characterised materials using the new model.

REFERENCES

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