Conservative Particle System

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ABSTRACT

Over recent decades, various mesh-free particle methods and their improvement approaches have been proposed and successfully applied for numerical analysis of continuum dynamics and others. Today the mesh-free particle methods play an indispensable role in the engineering applications; however, the study of their discretization procedures and properties are inadequate since they are still in the early days of development compared to the mature mesh-based numerical methods such as finite element, finite difference, and finite volume method. Some criteria are crucial to the numerical algorithm: consistency, stability, convergence (and its rate), conservation of the invariants, boundedness/monotoneness, computational efficiency, and so on. Note that these standards are not independent. In this study, we take particular note of compatibility between the consistency conditions of the discretization schemes and the conservation laws of the physical invariants at the discrete level (e.g. linear momentum, angular momentum, and total energy of system).

Some discrete particle system of typical strong-form mesh-free particle methods as we know them today preserve the important physical invariants — linear momentum, angular momentum, and total energy of the closed system. For example, specific formulation (i.e. skew(anti)-symmetric form of gradient) of the Smoothed Particle Hydrodynamics (SPH) method [1,2], the Hamiltonian SPH method [3], the Hamiltonian Moving Particle Semi-implicit (Hamiltonian-MPS) method for elastodynamics [4] and incompressible flow dynamics [5], and the state-based peridynamics [6] method are available; however, their discretized systems are not consistent or are at most first order consistent. In other words, the compatibility between 2nd and higher order spatial discretization consistency and the conservation of linear momentum, angular momentum, and total energy are not ensured.

In this study, we propose a methodology to ensure the compatibility between the higher order consistency conditions and the conservation laws of physical invariants at discrete level.

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