Enforcing differentiation/integration compatibility in meshless methods with nodal integration

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ABSTRACT

Since its first introduction in 1977 by Lucy ([1]) and Gingold and Monaghan ([2]) the intuitive but somewhat simple approach behind SPH method has been supplemented with different *correction procedures* in order to improve its performance.

Several authors ([3], [4], [5], [6], just to name a few) have indeed proposed some modifications of the discrete gradient that allow to recover at least linear consistency.

However the SPH method suffers from another weakness related this time to the numerical integration and which has recently been cast as *variational inconsistency* by Chen & al ([7]). This limitation prevents the method as such to pass the well-known *patch tests* in linear elasticity simulation and exhibits suboptimal convergence rate.

A few authors have proposed strategies as remedies for this limitation but they come with their own set of problems. Indeed, in ([8]) the corrective procedure requires the solving of a huge linear system, which scales with the number of edges in the point cloud. In ([9]) the proposed corrective system scales this time with the number of nodes but little is said on its solvability, whereas in ([7]) the procedure, although being local, breaks symmetry by introducing a Petrov-Galerkin formulation and, more importantly, raises significant questions about stability.

The present paper will address this question as well, formally introducing the concept of *differentiation/integration compatibility* as a discrete counterpart of the gradient theorem and analysing the consequences of its non-fulfilment in case where a weak formulation (e.g. Renormalized SPH) or a strong formulation (e.g. FPM- Finite Pointset Method) is employed.

A new procedure to recover compatibility will be introduced, which yields a symmetric positive definite linear system that scales with the number of points. Its solvability will be discussed and an efficient preconditioner will be proposed.

Strategies to reduce the CPU time spent in the correction procedure will finally be discussed based on numerical tests on some academic examples.

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