

Lagrangian Particle Method for Compressible Fluid Dynamics

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Abstract

A new Lagrangian particle method is proposed for solving Euler equations for compressible fluid flow. By representing Lagrangian fluid cells with particles, similarly to smoothed particle hydrodynamics (SPH) [1], the method eliminates the mesh distortion problem of the original Lagrangian method and is suitable for the simulation of complex free surface flows. The main contributions of our method, which is different from SPH in all other aspects, are (a) elimination of the dependence on artificial parameters such as the smoothing length in SPH, causing difficulties especially in the case of large density changes, and (b) significant improvement of approximation of differential operators and convergence of prescribed order. The major drawback of SPH is a very poor accuracy of discrete differential operators. It is widely accepted [2,3] that the traditional SPH discretization has zero-order convergence for widely used kernels. In addition, it depends on artificial parameters, in particular on the smoothing radius, causing major difficulties in the case of large density changes. A number of “modern” or “corrected” SPH methods have been developed in recent years (see [2] and reviews [1,3]). They include the moving-least-squares MLS-SPH, “Godunov” SPH, P_SPH, PHANTOM etc. They all improve certain features of SPH at the expense of other properties such as conservation, long-time stability, or prohibitively large number of neighbors that causes other problems, but still remain zero-order convergent, or at best 1st order (MLS).

The proposed Lagrangian particle method is significantly different from previous meshless methods. The novel key invention of our method is a stable particle-based upwinding method [4], capable of achieving a prescribed order of accuracy by employing a local optimization / polynomial fit (known also as the generalized finite difference (GFD) method) for the calculation of local gradients. A Riemann problem based algorithm for free surfaces has also been developed. The second order method was tested in one-, two-, and three spatial dimensions. It demonstrated a significant improvement of accuracy and convergence order compared to the traditional SPH. The future development of the space-time discretization methods will explore new high resolution WENO-type solvers based on irregularly placed particle nodes and symplectic integrators.

Our Lagrangian particle method, currently developed for compressible fluids, is also generalizable to coupled multiphysics systems, including the dynamics of incompressible fluids, solids, and plasmas. By generalizing this method to elliptic problems, we have recently proposed an optimal meshless solution to the classical particle-in-cell (PIC) problem that exceeded the accuracy and performance of PIC.

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