Effect of the separated approximation of input data in the accuracy of the resulting PGD solution

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ABSTRACT

Separated representations are commonly used to avoid the so called *curse of dimensionality*, that is, an exponential increase in the number of degrees of freedom with the number of dimensions.
In order to use separated functions to approximate the solution of some Boundary Value Problem (BVP), the differential operators defining the BVP must be separable. This is the case, for example, of the Proper Generalised Decomposition (PGD). Separability of operators, in practice, requires separability of input data. For example, the diffusivity function in the bilinear operator in a Poisson equation must be separable. In the case the original data is not separable, it has to be approximated by a separated *approximation*. This approximation can be obtained by means of a singular value decomposition, a proper orthogonal decomposition, a higher-order singular value decomposition, or other similar techniques.

Thus, in addition to the classical error sources in the PGD context, namely the PGD truncation and the underlying finite-element discretisation, the error arising from using a separable approximation of the input data is also affecting the PGD solution.

In this work we analyse numerically the error in the PGD solution due to the separation of the input data. This error is mainly controlled by two factors: (i) the mesh of the parametric dimensions (or parametric sampling) and (ii) the number of terms in the separated approximations of the input data. This source of error is eventually limiting the convergence or the solution because the data resolution establishes a threshold in the accuracy of the final approximation. In the problems analysed, these sources of errors are polluting the final result in a stable manner, that is, the error in the PGD solution can be controlled if the accuracy of the separated approximation of input data is small enough.