Particle Swarm Optimization Algorithms for Solving Large Systems of Nonlinear Equations

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ABSTRACT

The solution of nonlinear systems of equations is one of the most challenging problems of numerical mathematics [1], and the difficulties associated with it are amplified as the number of equations increases. However, it is not hard to find examples of systems with a relatively small number of nonlinear algebraic or transcendental equations, from different areas of science and engineering [2], including computational mechanics, which are difficult to solve by traditional numerical techniques.

Systems of nonlinear equations are of fundamental importance in many areas, such as Chemistry, Physics, Economics and different branches of Engineering [3], and usually appear as part of the simulation of physical processes [4], as in the case of physical models expressed by nonlinear partial differential equations, whose discretization leads to large systems of nonlinear equations.

Although there are a variety of iterative methods in the literature for the numerical solution of systems of nonlinear equations, much of them are severely limited by their domains of convergence (e.g., most of the Newton-like methods for solving nonlinear equation systems are only locally convergent [5]), being the success of their application strongly dependent on the initial approximations used. The cases of non-convergence to any of the problem solutions are relatively frequent in practice and unfortunately, there does not exist a computationally efficient and numerically robust method of general applicability for the solution of this class of problems [6].

Due to this drawbacks, in more recent years metaheuristic and hybrid approaches [7] have found increasing use for solving nonlinear equation system problems [8, 9]. These approaches generally consist in transforming the original problem into a nonlinear optimization problem, and numerically approximating a solution to this problem by using a pure metaheuristic search algorithm or a hybrid metaheuristic-based strategy [8-12].

In this study, the application of particle swarm optimization (PSO), a population-based metaheuristic [13], is considered for the problem of solving large systems of nonlinear equations, focusing the performance impact of different variants of the basic PSO algorithm, including constant, time varying and adaptive parameter strategies with different swarm topologies. The PSO variants considered in this study were evaluated and compared by using a set of 50 test problems from the literature with different ranges of difficulty. For each test problem, 100 independent trials were run with each PSO algorithm. The quality of the generated approximations, the computational efficiency, and the number of successes in the independent trials were used as basic evaluation criteria for these variants of PSO.

The obtained results provide information on the influence of the different parameter strategies and swarm topologies analysed over the performance and relative dominance of the resulting PSO variants in solving large systems of nonlinear equations.

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