Comparison of different FFT-based methods for computing the mechanical response of heteregoneous materials

H. Moulinec*, F. Silva[1], P. Suquet[2]

CNRS LMA,
31 chemin Joseph Aiguier 13402 Marseille cedex 20 FRANCE
* moulinec@lma.cnrs-mrs.fr
[1] silva@lma.cnrs-mrs.fr
[2] suquet@lma.cnrs-mrs.fr

The last decade has witnessed a growing interest for the so-called "FFT-based methods" for computing the overall and local properties of heterogeneous materials submitted to mechanical solicitations. Since the original method was introduced by Moulinec and Suquet [1], several authors have proposed different algorithms to better deal with non-linear materials or with materials with highly contrasted mechanical properties between their constituents.

The study concerns a linear elastic material - although the methods involved can be extended into the case of non-linear behavior - submitted to a prescribed overall strain E. The stiffness tensor $c(x)$ of the material varies with the position x. The numerical method proposed by Moulinec & Suquet lies on the iterative resolution of the Lippmann-Schwinger equation and can be summarized by the following relation between two successive iterates ε^{i} and ε^{i+1} of the strain field:

$$\varepsilon^{i+1}(x) = -\Gamma^0 * \Big((c(x) - c^0) : \varepsilon^{i}(x) \Big) \quad + E \quad ,$$

where c^0 is the stiffness tensor of a reference medium supposed to be linear elastic, where Γ^0 is a Green operator associated to c^0 and where $*$ denotes the convolution operator.

Eyre & Milton [2], Michel et al. [3] and Monchiet & Bonnet [4] proposed different schemes to accelerate the convergence of the initial scheme. It has been recently demonstrated in [5] that the two first schemes are particular cases of the last one. On the other hand, Zeman et al. [6] proposed to use a conjugate gradient method for solving the Lippmann-Schwinger equation.

The present paper aims to compare these different methods with a special attention paid to their relative efficiency and their rates of convergence.

References

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