Optimal preconditioning of adaptive FEM-BEM couplings

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ABSTRACT

For many relevant applications, the coupling of the finite element method (FEM) and boundary element method (BEM) appears to be the appropriate numerical method to cope with unbounded domains. As the problem size increases, so does the strong need for effective preconditioners for iterative solvers. Most of the available literature on preconditioning of FEM-BEM coupling techniques deals with the symmetric coupling on quasi-uniform meshes. Often, however, non-symmetric coupling formulations are preferred, since they avoid the computation and evaluation of the hypersingular integral operator.

We present results [1] on block-diagonal preconditioning for the Johnson-Nédélec coupling on adaptively generated meshes. We consider block-diagonal preconditioners, where the two diagonal blocks are symmetric and positive definite. These are obtained from a local multilevel additive Schwarz decomposition of the energy space. While the analysis relies on this abstract frame, the resulting preconditioners are obtained from simple algebraic postprocessing of the (history of the) Galerkin matrix.

Starting from an initial mesh which is adaptively refined by bisection, we prove that the condition number of the preconditioned system remains bounded, where the bound depends only on the initial mesh.

Although we shall mainly discuss the 2D Laplace transmission problem, the principal ideas also apply to the 3D case and Lamé-type transmission problems. Moreover, the analysis transfers to other coupling methods, such as the symmetric coupling or the symmetric and non-symmetric Bielak-MacCamy coupling.

REFERENCES

[1] M. Feischl, T. Führer, D. Praetorius, E.P. Stephan, “Optimal preconditioning for the symmetric and non-symmetric coupling of adaptive finite elements and boundary elements”, *ASC Report*, **36/2013**, (2013).