High Order Shape Sensitivity Analysis of Weighted Residual Methods

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ABSTRACT

On a regular basis, engineering analysis requires stating and solving systems of partial differential equations (PDEs). The most powerful and widely extended techniques for solving PDEs are the so-called Weighted Residuals Methods. To this group belong, among others, the Finite Element Method (FEM), the Boundary Element Method (BEM), the Finite Volume Method (FVM) and the Mesh-Free Method (MFM), as well as the many different formulations included in each of these categories. The new Isogeometric Analysis Methods (IGA) were proposed in 2005 [1], and it is our belief that they really deserve special attention.

The sensitivity analysis [2] of all the above mentioned formulations requires taking derivatives of functions that are defined by integration in arbitrary domains. If the geometry of the problem being solved is constant, so will be the integration domains. In these conditions it is fairly straightforward to state the sensitivity analysis by means of fully analytical techniques. On the contrary, if the geometry of the problem being solved is not constant (i.e. in shape optimization or shape parameter estimation problems, non-confined flow problems, etc.) stating the sensitivity analysis is not immediate. For this reason, the sensitivity analysis for varying-geometry problems has been mainly addressed by means of low order finite difference approximations, which are known to be inaccurate and difficult to calibrate. In an attempt to overcome these drawbacks, a number of analytical approaches have been proposed to address the sensitivity of 1) the numerical implementation, 2) the analytical model in weak form, and 3) the discretized formulation corresponding to the numerical method being used [3].

In our presentation we will state a general formulation for the sensitivity analysis of Weighted Residual Methods, both for linear and non-linear problems with constant or varying geometry. The effects due to variation of geometry are addressed by defining a generic procedure for integration in manifolds on the basis of the metric tensor concept. The proposed approach leads to compact and relatively simple expressions to obtain directional derivatives of arbitrarily high order. The resulting scheme can be easily applied to FEM and IGA formulations, being its implementation a quite straightforward task. Finally, some application examples will be presented.

REFERENCES

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