Probability Estimation of Uncertain Linear Dynamical Systems

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ABSTRACT

In applications of engineering interest, there is often uncertainty on the performance of a structural system during its lifetime. Two typical sources of uncertainty refer to the loading the system undergoes and the actual parameters characterizing its structural properties. For such cases, stochastic processes and random fields allow modeling the uncertainty associated with loadings and properties, respectively. For the particular case of structural systems subject to dynamic excitations, a means for taking into account the effect of uncertainty and measuring safety is the so-called first excursion probability, i.e. the probability that one or more estructural responses exceed a prescribed threshold within the duration of a stochastic excitation. Although first excursion probability provides much information on the safety of the structure, its calculation is extremely challenging, as it usually comprises a large number of both failure criteria and random variables (required for representing in a discrete way stochastic processes and random fields). In fact, integrals associated with first excursion probabilities usually involve hundreds or even thousands of dimensions. Hence, this contribution proposes an efficient strategy based on simulation for evaluating such probabilities. More specifically, this contribution focuses on uncertain linear systems subject to stochastic excitation. Uncertainty on structural properties is characterized by means of homogeneous log-normal random fields while uncertainty on excitation is characterized as discrete white noise.

The proposed strategy is based on simulation and applies Importance Sampling. For this purpose, two Importance Sampling density functions (ISD) are applied, which are related to the uncertain excitation [1] and uncertain structural parameters [2], respectively. Samples of the uncertain structural parameters (distributed according to the prescribed ISD function) are generated using the so-called Sampling-Importance Resampling (SIR, [3]) algorithm. In particular, a SIR algorithm with replacement is considered which is used together with a surrogate model for determination of spectral properties. The surrogate model considered in this contribution (which was developed in [4]) combines two basic ideas: an interpolation strategy and the application of a reduced basis; some variants of this surrogate model are evaluated in order to decrease numerical cost. The efficiency and advantages of the proposed are illustrated by means of a numerical example.

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