A Finite Volume Method for the Mixture Model of Incompressible Multiphase Flows

Santiago Márquez Damián[1]*, Norberto M. Nigro[1] and Gustavo C. Buscaglia[2]

[1]Centro de Investigaciones en Mecánica Computacional, (CIMEC) UNL/CONICET
Colectora Ruta Nac. 168 / Paraje El Pozo, (3000) Santa Fe, Argentina
`http://www.cimec.org.ar`

[2]Instituto de Ciências Matemáticas e de Computação (ICMC), USP
Av. do Trabalhador Sãocarlense 400, Centro, CEP 13560-970, São Carlos, SP, Brasil,
`http://www.icmc.usp.br`

ABSTRACT

This work presents a Finite Volume method for the solution of the Mixture Model equations [1] in a mixed formulation written in terms of $\vec{v}$ (mass-averaged velocity), p (mixture pressure) and $\underline{\alpha} \in \mathbb{R}^d$ (array of volume fractions of the d disperse phases) as

$$\begin{cases} \vec{\nabla} \cdot \vec{u}(\underline{\alpha}, \vec{v}) = 0 & \text{(a)} \\ \partial_t(\rho(\underline{\alpha})\vec{v}) + \vec{\nabla} \cdot (\rho(\underline{\alpha})\vec{v} \otimes \vec{v}) = -\vec{\nabla} p + \rho(\underline{\alpha})\,\vec{g} + \vec{\nabla} \cdot \mathbb{T} & \text{(b)} \\ \partial_t \underline{\alpha} + \vec{\nabla} \cdot \left(\underline{\underline{\vec{V}}}(\underline{\alpha}, \vec{v}) \;\; \underline{\alpha} \right) = \underline{S} & \text{(c)} \end{cases} \tag{1}$$

where the auxiliary variables $\vec{u}$ (volume-averaged velocity), ρ (mixture density), and $\underline{\underline{\vec{V}}}$ (transport velocity matrix) are explicit functions of the primary unknowns. The tensor $\mathbb{T}$ models viscous effects and momentum exchange between the phases, and $\underline{\underline{\vec{V}}}$ is a diagonal matrix with the transport velocity of each disperse phase as its diagonal entries.

The discretization by the Finite Volume Method requires the calculation of fluxes for both $\vec{v}$ and $\vec{u}$, and transformation formulas (back and forth) that enforce mass conservation are presented, based on the work of Bohorquez [2]. Also addressed is the integration of the mass conservation equations for the disperse phases (1.c), which may exhibit shocks and rarefaction waves due to the nonlinearity of the flux function. This task is achieved by a Riemann-free solver based on the Kurganov and Tadmor scheme [3]. The proposed method has been implemented with the OpenFOAM® libraries and tested on a series of examples. Comparisons with analytical and experimental results in problems involving 1D waves, bubble plumes and inclined sedimentation provide evidence of good performance in cases with a single disperse phase ($d = 1$). The communication ends with a discussion of polydisperse ($d > 1$) case [4].

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