

Nonlocal models of anomalous transport in bounded domains

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ABSTRACT

Nowadays it is well recognized that the extensively used Fourier-Fick, flux-gradient local prescription might break down in systems exhibiting anomalous diffusion. Examples of particular interest to this presentation include turbulent and chaotic transport in fluids and plasmas. Nonlocal transport models provide an alternative to the Fourier-Fick prescription by allowing the flux to depend on the global spatio-temporal properties of the transported field. Formally, in these models the flux-gradient involves integro-differential operators with spatial kernels to account for nonlocality in space and temporal kernels to account for nonlocality in time (i.e., memory effects). In this talk we will limit attention to nonlocality in space and consider flux-gradient relations involving fractional derivatives. In the simplest case of constant transport coefficients and unbounded domains, the nonlocal model reduces to the extensively studied and well-understood fractional diffusion equation. However, the study of nonlocal transport in physically relevant systems requires the formulation of mathematically well-posed and physically meaningful nonlocal models in bounded spatial domains. The main problem faced by fractional diffusion models in this case resides in the treatment of the boundaries. For example, the naive truncation of the Riemann-Liouville fractional derivative in a bounded domain is in general singular at the boundaries and, as a result, the incorporation of generic, physically meaningful boundary conditions is not feasible. In this presentation we discuss alternatives to address the problem of boundaries in fractional diffusion models. Our main goal is to present models that are both mathematically well posed and physically meaningful. Our approach is based on the regularization of the singularities at the boundary. Following the formal construction of the models in 1-dimensional and 2-dimensional bounded domains, we present finite-difference methods to solve the regularized models, and conclude with examples of nonlocal heat transport in nuclear fusion plasmas.