Discrete optimality in nonlinear model reduction: analysis and application to computational fluid dynamics

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ABSTRACT

Many time-critical applications in science and engineering (e.g., model predictive control, Bayesian inference) demand the accuracy of high-fidelity physics-based models, yet cannot afford their computational cost. To reduce this computational burden, a variety of reduced-order modelling (ROM) techniques have been developed. These methods can be categorized as exhibiting *continuous optimality* or *discrete optimality.* Continuous-optimal ROMs perform a projection process that leads to an optimality property at the time-continuous (i.e., ordinary differential equation) model; such ROMs dominate the literature, and include the ubiquitous proper orthogonal decomposition (POD)–Galerkin method. On the other hand, discrete-optimal ROMs employ a projection process that results in an optimality property at the time-discrete (i.e., algebraic) level. This class of methods is far less common, and includes residual-minimizing techniques [1] such as the Gauss–Newton with Approximated Tensors (GNAT) technique [2,3].

To date, the relationship between continuous- and discrete-optimal ROMs has not been adequately characterized. For example, the expression of discrete-optimal ROMs as a low-dimensional (time-continuous) ODE has not been shown. Further, the performance of discrete-optimal ROMs as a function of discretization parameters has been largely unexplored.

In this work, we provide a consistent analysis framework to relate these two classes of reduced-order models. This analysis reveals equivalence of the two classes of ROMs in certain limits. In addition, we characterize the error of discrete-optimal ROMs and show that the error bound depends strongly on specific discretization parameters that are often overlooked or ignored in the context of model reduction. We also provide a mechanism for selecting such parameters to improve the accuracy and stability of discrete-optimal ROMs. Finally, we demonstrate the practical implications of these analytical results on a large-scale, unsteady problem in computational fluid dynamics characterized by over one million unknowns.

REFERENCES

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