The h-version of the method of auxiliary mapping for higher order solutions of crack problems

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High order finite element solutions of crack an re-entrant corner problems in elasticity are notoriously challenging to attain. The slow convergence rates for this classes of problems are tied to the low degree of regularity possessed by the solution. The need for high order solutions is motivated by long running simulations of crack propagation. In fact, as the error in the crack path grows non-linearly, it is of uttermost importance to accurately resolve the elasticity fields at each time step. The above ensures that the final computed crack paths are within reasonable tolerance of the exact one.

To this extent, we develop higher order finite element methods for crack and re-entrant corner problems in elasticity. The framework is cast within the context of conforming finite elements [3]. The method [2] exploits the a priori knowledge of the singular behavior of the fields to construct an alternate regular solution. Solving for the alternate problem yields optimal rates of convergence and high order of accuracy. Namely, standard finite element error estimates in the L^2 and H^1 norm are proved to hold, both for the alternate solution and the composed alternate solution. The salient feature of the method is the lack of additional degrees of freedom in comparison with its standard Galerkin finite element formulation. Effectively for the same computational cost we obtain a higher order of accuracy. Furthermore, unlike other state of the art tools, the method preserves the well conditioned nature of the system of equations and does not require the knowledge of the exact asymptotic behavior.

The method is verified with respect several analytical solutions in two-dimension and three-dimensions. Two dimensional examples account for the possible curvilinear behavior of fractures.

Along with the above we employ interaction integrals for curvilinear fractures as presented in [1] and generalize their definition for the proposed higher order method. Along with the optimality of the convergence of the solution we showcase the accuracy and the convergent behavior of the computed stress intensity factors.

The application of the framework are showcased for complex fracturing problems. In particular, simulations of fracture instabilities in thermoelastic materials subjected to large temperature gradients, where oscillatory fracture behavior is expected, will be used to demonstrate the robustness and capabilities of the presented tools.

References

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