Varying the Block Size on Block Conjugate Gradient: Comparison of Strategies

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ABSTRACT

The Conjugate Gradient (CG) method has been widely used to solve linear systems of the type when the matrix A is symmetric positive definitive. In the context of the implementation of CG as an iterative algorithm one important bottleneck is in the matrix vector multiplication which has the disadvantages of a low ratio of computation-memory accesses. To improve this ratio several strategies have been researched ranging from new paradigm machine architectures to optimizing the cache data usage through the usage of a matrix multivector multiplications Kernel. The latter in the CG context yields the Block Conjugate Gradient (BCG) originally used to solve linear systems with multiple right-hand-side. A main drawback of BCG for the case of a single right hand side is the determination of an optimal block size. Moreover, if an adequate block size is not chosen the effect on the rate of convergence of the CG can be negative and consequently its rate of convergence deteriorates. In this work we compare strategies for determining adaptively the optimal block size in the implementation of BCG considering a single right hand side seeking a trade-o_ between convergence rate and computational cost. The strategies analyzed are based on the Rayleigh Values associated to the search subspace, the associated Ritz Values, and the decay rate of the residual measured locally. Numerical experiments on real matrices are presented. It is observed that the Ritz Values strategy shows better results than the other strategies in terms of decreasing the computational cost. These results encourage further analysis and design of adaptive block algorithms.

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