Ritz–Volterra reconstructions and *a posteriori* error analysis of finite element method for parabolic integro-differential equations

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ABSTRACT

Parabolic integro-differential equations (PIDE) arise in various applications. Some of its occurrence includes heat conduction in material with memory, the compression of poro-viscoelasticity media, nuclear reactor dynamics and the epidemic phenomena in biology. Despite being so rich in the *a priori* analysis and in spite of the importance of these equations in the modelling of several physical phenomena, the topic of *a posteriori* error analysis for such kind of equations remains unexplored. Since PIDE may be thought of as a perturbation of the purely parabolic problem, therefore, it is natural to see whether the *a posteriori* error analysis of parabolic problems can be extended to PIDE. An attempt has been made in this work to generalize the results of purely parabolic problems to PIDE. A *posteriori* error estimates for both semidiscrete and implicit fully discrete backward Euler method for linear parabolic integro-differential equations are obtained in a bounded convex polygonal or polyhedral domain. A novel space–time reconstruction operator is introduced, which is a generalization of the elliptic reconstruction operator [*SIAM J. Numer. Anal.*, **41**, pp. 1585–1594, (2003)], and we call it as Ritz–Volterra reconstruction operator. The Ritz–Volterra reconstruction operator in conjunction with the linear approximation of the Volterra integral term is used in a crucial way to derive optimal order *a posteriori* error estimates in $L_\infty(L_2)$ and $L_2(H_1)$-norms. The related *a posteriori* error estimates for the Ritz-Volterra reconstruction error are also established. It is observed that the Ritz–Volterra projection is useful in *a priori* analysis for a wide range of (linear and nonlinear) parabolic and hyperbolic integro-differential problems. We strongly believe that, the Ritz–Volterra reconstruction operator, a counterpart of the Ritz–Volterra projection in the *a priori* analysis, can be appropriately modified to obtain estimators for a class of integro-differential problems. Moreover, Ritz Volterra reconstruction operator unifies *a posteriori* approach from parabolic problems to PIDE.

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