Dynamic Viscosity Formulation for Steady State and Transient Advection-Diffusion-Reaction Problems

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ABSTRACT

In this work, we evaluate a nonlinear multiscale model [1,2,3] for solving steady and transient advection-diffusion-reaction equations using first-order and second-order time approximation schemes. The present multiscale method, named Dynamic Diffusion method (DD), adds to the Galerkin formulation in a consistent way a nonlinear dissipative operator, which is isotropic and is scaled by an eddy viscosity. It results in a free parameter method in which the eddy viscosity is evaluated locally and dynamically, depending only on the accuracy of the resolved scale solution [1]. In order to reduce the computational cost typical of multiscale methods, continuous finite elements are considered and the small scale space is defined using bubble functions whose degrees of freedom are condensed onto the resolved scale degrees of freedom [4]. Here we use iteration-lagging for the nonlinear procedure in the space domain and the resulting system of ordinary differential equations is solved by finite difference schemes considering static and temporal variation of the subscales. In the static approach, we assume that the subscales do not change in time, which yields a quasi-static (QS) method. In [4] we illustrated the behavior of QS and two different time schemes, named finite difference and predictor-multicorrector (FDPM) scheme and backward Euler (BE) method. Here, we consider also a second order time approximation scheme based on a predictor-multicorrector (PM) method. Numerical experiments for standard problems are presented, and the accuracy and convergence of the proposed methodologies are examined for new characteristic sub-grid parameters, such as the subgrid velocity field and the length scale at which the subgrid inertial effects take place.

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