

# DISCONTINUOUS PETROV-GALERKIN (DPG) METHOD WITH OPTIMAL TEST FUNCTIONS FOR SPACE-TIME DISCRETIZATIONS OF COMPRESSIBLE NAVIER-STOKES EQUATIONS

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**Abstract.** The Discontinuous Petrov-Galerkin Method (DPG) guarantees a stable discretization for any well-posed problem [1, 2]. This makes it especially attractive in context of singular perturbation problems [3, 4, 5] for which the standard Galerkin method fails and where one strives for the construction of *robust* discretizations, i.e. discretizations that are uniformly stable in the perturbation parameter.

The presentation will cover selected results on the development of a DPG method for space-time discretizations of compressible Navier-Stokes equations in the large Reynolds number regime. The methodology is based on the first order (ultra-weak) formulation and extrapolation of rigorous robustness analysis for convection-dominated diffusion in space-time. Entropy function and the corresponding symmetrizers are used to define norms in which we strive for the robustness.

We will state the main theoretical results and illustrate them with 1D and 2D (in space) examples of adaptive solutions.

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