Impedance Functions for Axial Symmetric Foundation in Layered Half-space by Fundamental Solution Method

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The presentation will be a procedure to generate impedance functions for axial symmetric foundation embedded in layered half-space. In the procedure, the fundamental solution for a triangularly distributed loading on an axial symmetric region of any horizontal plane in layered half-space will be introduced. Since the loading is distributed on an axial symmetric region, no singularity like Green Function will occurs. The Fundamental Solution will give the relationship between traction vector and displacement vector along the interface \( s \) of foundation and surrounding medium.

Therefore, the approximate stress field and displacement field along the interface can be expressed as
\[
\mathbf{t}(s) = \sum_{i}^{n} M_i P_i = \mathbf{MP} \quad \text{and} \quad \mathbf{u}(s) = \sum_{i}^{n} N_i P_i = \mathbf{NP}
\]
respectively, where \( M_i \) and \( N_i \) are the \( i^{th} \) fundamental solution for traction vector and displacement vector, respectively, \( n \) is the total number of fundamental solutions employed, and \( P_i \) is the corresponding unknown parameter for \( i^{th} \) fundamental solution. If \( \mathbf{N} \) is the shape function matrix for prescribed displacement function of foundation, the prescribed displacement of the foundation on interface \( s \) can be expressed as \( \mathbf{u}(s) = \mathbf{Nu}_f \), where matrix \( \mathbf{u}_f \) is the nodal displacements at interface \( s \). The nodal force matrix \( \mathbf{F}_f \) at the interface \( s \) must have the form
\[
\mathbf{F}_f = \int_{A} \mathbf{N}^T \mathbf{t}(s) ds.
\]
Using weighed residual method, one obtains
\[
\int_{A} \delta \mathbf{t}(s) \left[ \mathbf{u}(s) - \mathbf{u}(s) \right] ds = 0,
\]
where \( A \) is the area of interface \( s \). After some mathematical manipulations, the impedance matrix \( \mathbf{I} \) can be obtained as
\[
\mathbf{I} = \mathbf{k}_c^T \mathbf{k}_c^{-1} \mathbf{k}_e, \quad \text{where} \quad \mathbf{k}_c = \int_{A} \mathbf{M}^T \mathbf{N} ds \quad \text{and} \quad \mathbf{k}_e = \int_{A} \mathbf{M}^T \mathbf{N} ds.
\]

Some numerical results will be presented. The precision of results and efficiency of the method will be investigated. Some conclusions will be made.