Neumann Solutions to Fractional Lamé-Clapeyron-Stefan Problems with Heat Flux or Convective Boundary Conditions

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ABSTRACT

In the last decades the fractional differential equations were developed [2, 3, 4] and in the recent years some works on the fractional Lamé-Clapeyron-Stefan problem were published [1, 5, 6, 9, 10].

In this paper, generalized Neumann solutions for the two-phase fractional Lamé-Clapeyron-Stefan problems for a semi-infinite material are obtained with constant initial condition, and a boundary condition at the fixed face $x = 0$ given by: a heat flux or a convective (Robin) condition.

In these problems, the two governing diffusion equations and a governing condition for the free boundary include a fractional time derivative in the Caputo sense of order $0 < \alpha < 1$. When $\alpha \to 1^-$ we recover the classical Neumann solutions for the two-phase Lamé-Clapeyron-Stefan problem through the error function, given in:

(i) [7] for a heat flux boundary condition at the fixed face $x = 0$ when an inequality for the coefficient which characterizes the heat flux boundary condition is satisfied;
(ii) [8] for a convective boundary condition at the fixed face $x = 0$ when an inequality for the coefficient which characterizes the convective boundary condition is satisfied.

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