

Polynomial chaos-based metamodeling for solving inverse heat conduction problems

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ABSTRACT

Inverse heat conduction problems (IHCPs) constitute an interesting class of inverse problems that is of importance in many branches of science and engineering. In recent years Bayesian inference has been established as a powerful framework for inversion and uncertainty quantification in general [1,2] and for solving IHCPs in particular [3,4]. While the Bayesian approach to inverse problems is conceptionally straightforward, the practical computation of the posterior can be prohibitively expensive. Sampling the posterior via Markov chain Monte Carlo (MCMC) techniques usually requires a fairly high number of forward model runs.

In order to accelerate Bayesian inference it has been proposed to replace calls to the forward model by calls to an inexpensive surrogate such as a polynomial chaos expansion (PCE) [5,6]. This approach promises drastic speedups. When a “good” approximation of the forward model is employed, one can expect a “good” approximation of the posterior that is computed this way.

In this contribution we consider an IHCP based on the stationary heat equation. Particularly we consider the problem of inferring different unknown thermal conductivities of a composite material with close-to-surface temperature measurements. We show how the determination of thermal material properties can be substantially accelerated by employing PCEs. Firstly we will investigate the use of prior-based PCE-surrogates of the forward model, i.e. expansions in polynomials that are orthogonal with respect to the prior distribution. Secondly we will investigate the applicability and performance of metamodeling the log-likelihood function for IHCPs.

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